Microeconomics (5SSPP217)

Seminar 0

Felipe Torres
felipe.torres@kcl.ac.uk

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Contact & Office Hours

- My email: **felipe.torres@kcl.ac.uk**
- Office hours: Every Wednesday from 13:00 until 14:00. This week: Bush House BH (NE)7.22.
- Only this week: Tomorrow at BH (NE)8.02 from 9:30 until 10:30
- I will send you the slides once the solution sheet is made available on KEATS.

Question 1, a

1. A firm selling beds is a price taker. The current market price of beds is 200. The firm's total costs are given by the equation:

$$C(q) = 100 + 160q + 0.5q^2 \tag{1}$$

Where q is the number of beds produced (and sold).

a) What is the firm's optimal production level? How much profits does it earn?

Question 1, a

1. Set up:
   - We are looking for q where the firm maximises its profits.
   - We need to know what is the rule that determines the firm's optimal level of production.

Question 1, a

In a perfect competitive market, the firm will produce when the market price is equal to the Marginal Cost (MC)

$$P = MC \tag{2}$$

$$MC = \frac{\partial C(q)}{\partial q} \tag{3}$$

$$C(q) = 100 + 160q + 0.5q^2 \tag{4}$$

$$MC = \frac{\partial C(q)}{\partial q} = \frac{\partial 100}{\partial q} + \frac{\partial 160q}{\partial q} + \frac{\partial 0.5q^2}{\partial q} \tag{5}$$

$$MC = \frac{\partial C(q)}{\partial q} = 0 + 1 \cdot 160q^{1-1} + 2 \cdot 0.5q^{2-1} \tag{6}$$

$$MC = 0 + 160 + q \tag{7}$$

Question 1, a

$$MC = 160 + q \tag{8}$$

We know that the firm maximise determines its optimal production level when $MC = P$

$$MC = P \Rightarrow 160 + q = 200. \tag{9}$$

$$q^* = 40 \tag{10}$$

Question 1, a

a) How much profit does it earn?

Question 1, a

We need to set up the profit function, which is total revenue minus total cost:

$$\pi = TR - TC \tag{11}$$

$$\pi = p \cdot q - C(q) \tag{12}$$

$$\pi = p \cdot q^* - (100 + 160 \cdot q^* + 0.5 \cdot q^2) \tag{13}$$

$$\pi = 200 \cdot 40 - (100 + 160 \cdot 40 + 0.5 \cdot 40^2) \tag{14}$$

$$\pi = 8,000 - 7300 \tag{15}$$

$$\pi^* = 700 \tag{16}$$

Question 1, b

1. A firm selling beds is a price taker. The current market price of beds is 200. The firm's total costs are given by the equation:

$$C(q) = 100 + 160q + 0.5q^2 \tag{17}$$

Where q is the number of beds produced (and sold).

b) Suppose that fixed costs increase from their original level to 100 to 1000. How does this the firm's optimal decision?

Question 1, b

So now we have a new cost function. What do we have do now?

$$C(q) = 1000 + 160q + 0.5q^2 \tag{18}$$

Question 1, b

We have to do the same that we did before. Let's get MC, set it equal to the market price and let's look at the profits.

$$C(q) = 1000 + 160q + 0.5q^2 \tag{19}$$

$$\frac{\partial C(q)}{\partial q} = \frac{\partial C(q)}{\partial q} + \frac{\partial 1000}{\partial q} + \frac{\partial 160q}{\partial q} + \frac{\partial C(0.5q^2)}{\partial q} \tag{20}$$

$$\frac{\partial C(q)}{\partial q} = 0 + 160 + q \tag{21}$$

Question 1, b

So our marginal cost is the same:

$$MC = 160 + q \tag{22}$$

Thus:

$$MC = P \tag{23}$$

$$160 + q = 200 \tag{24}$$

$$q^* = 40 \tag{25}$$

Question 1, b

Let's get the profits with this Cost function

$$\pi = p \cdot q^* - (1000 + 160 \cdot q^* + 0.5 \cdot q^2) \tag{26}$$

$$\pi = p \cdot 40 - (1000 + 160 \cdot 40 + 0.5 \cdot 40^2) \tag{27}$$

$$\pi = -200 \tag{28}$$

Thus, the firm can avoid the fixed cost by existing the market, thus $q^* = 0, and \pi^* = 0$

Question 2

2. Find the stationary points of the function:

$$f(x, y) = y^2(1 - x) - x^2(1 + x) \tag{29}$$

Once you do that, construct the Hessian matrix and use it to classify each of the stationary points you found.

Question 2

2. Find the stationary points of the function:

How do we find the stationary points?

Question 2

2. Find the stationary points of the function:
   - First, we obtain the potential candidates of stationary points by getting partial derivatives w.t.r to x and y
   - We then create the Hessian matrix, which is the square matrix of second-order partial derivatives
   - We plug the potential candidates of stationary points in the Hessian matrix
   - We then calculate the determinant, which is the product of the eigen values inside of the matrix (more details later).
   - and with this we can determine whether each stationary point is a local maximum/minimum or a saddle point

Question 2

2. Find the stationary points of the function:

$$f(x,y) = y^2(1-x) - x^2(1+x) \tag{30}$$

$$\frac{\partial f}{\partial x} = \frac{\partial y^2(1-x)}{\partial x} - \frac{\partial x^2(1+x)}{\partial x} \tag{31}$$

$$\frac{\partial f}{\partial x} = y^2(-x^{1-1}) - \frac{\partial x^2(1+x)}{\partial x} \tag{32}$$

We need to use the product rule:

$$\frac{d}{dx}(f(x)g(x)) = f(x)\frac{d}{dx}g(x) + \frac{d}{dx}f(x)g(x) \tag{33}$$

$$\frac{\partial f}{\partial x} = -y2 - (\frac{\partial x^2}{\partial x}(1+x) + \frac{\partial(1+x)}{\partial x}x^2) \tag{34}$$

Question 2

2. Find the stationary points of the function:

$$\frac{\partial f}{\partial x} = -y^2 - (2x(1+x) + 1 \cdot x^2) \tag{35}$$

$$\frac{\partial f}{\partial x} = -y^2 - (2x + x^2) + x^2) \tag{36}$$

$$\frac{\partial f}{\partial x} = -y^2 - 2x - 3x^2 = 0 \tag{37}$$

Now let's continue with the derivative w.r.t to y.

$$f(x, y) = y^2(1-x) - x^2(1+x) \tag{38}$$

$$\frac{\partial f}{\partial y} = \frac{\partial y^2(1-x)}{\partial y} - y2 - \frac{\partial x^2(1+x)}{\partial y} \tag{39}$$

Question 2

2. Find the stationary points of the function:

$$\frac{\partial f}{\partial y} = 2 \cdot y^{2-1}(1-x) - 0 = 0 \tag{40}$$

$$\frac{\partial f}{\partial y} = 2y(1-x) = 0 \tag{41}$$

$$2y(1-x) = 0 \tag{42}$$

And we also have that:

$$y^2 - 2x - 3x^2 = 0 \tag{43}$$

Question 2

We also these two function[1]:

$$2y(1-x) = 0 \tag{44}$$

$$-y^2 - 2x - 3x^2 = 0 \tag{45}$$

If y = 0, then

$$0 - 2x - 3x^2 = 0 \tag{46}$$

$$-2x = 3x^2 \longrightarrow 3x = -2 \longrightarrow x = \frac{-2}{3} \tag{47}$$

[1] See in the appendix how to get the stationary points using substitution

Question 2

We also these two function:

$$2y(1-x)=0 \tag{48}$$

$$-y^2-2x-3x^2=0 \tag{49}$$

If $x = 1$, then:

$$y^2-2\cdot 1-3(1)^2=0 \longrightarrow -y^2-2-3=0 \longrightarrow . -y^2=5 \tag{50}$$

But there is no value for y^2 that will meet this equality since y^2 is always positive, but it's multiplied by -1. Thus, $x=0$ is not an option.

Question 2

We also these two function:

$$2y(1-x)=0 \tag{51}$$

$$-y^2-2x-3x^2=0 \tag{52}$$

If $x = 0$, then $y = 0$.
So we have two stationary points, $(0, 0)$ and $(\frac{-2}{3}, 0)$

Question 2

Let's get the Hessian and use it to determine whether our stationary points are minimum/maximum or saddle points.

Question 2

$$(\mathrm{Hess}\, f)_{ij} \equiv \frac{\partial^2 f}{\partial x_i \partial x_j} \tag{53}$$

$$\boldsymbol{H} = \begin{bmatrix} \frac{\partial^2 f}{\partial y^2} & \frac{\partial^2 f}{\partial xy} \\ \frac{\partial^2 f}{\partial yx} & \frac{\partial^2 f}{\partial x^2} \end{bmatrix} \tag{54}$$

Let's get the second derivatives.

Question 2

Just to refresh your memory:
With the Hessian, we are essentially conducting a second partial derivative test to identify whether each point is a stable local maximum or local minimum or a saddle point.

$$\boldsymbol{H} = f_{xx}(x_0, y_0) f_{yy}(x_0, y_0) - f_{xy}(x_0, y_0)^2 \tag{55}$$

- if the determinant of $\mathbf{H} < 0$, then (x_0, y_0) is a saddle point
- if the determinant of $\mathbf{H} > 0$, then (x_0, y_0) is a either a maximum or a minimum point
- if the determinant of $\mathbf{H} = 0$, then, we don't know what (x_0, y_0) is

Then, we need to check the second derivative of the variable of interest:

- if $f_{xx}(x_0, y_0) < 0$, then (x_0, y_0) is a local maximum
- if $f_{xx}(x_0, y_0) > 0$ then (x_0, y_0) is a local minimum

Question 2

Let's get the second derivatives:

$$\frac{\partial f}{\partial y} = 2y(1-x) \tag{56}$$

$$\frac{\partial^2 f}{\partial y^2} = 2y^{1-1}(1-x) \tag{57}$$

$$\frac{\partial^2 f}{\partial y^2} = 2(1-x) \tag{58}$$

Question 2

Let's get the second derivatives:

$$\frac{\partial f}{\partial x} = -y^2 - 2x - 3x^2 \tag{59}$$

$$\frac{\partial^2 f}{\partial x^2} = 0 - 1 \cdot 2x^{1-1} - 2 \cdot 3x^{2-1} \tag{60}$$

$$\frac{\partial^2 f}{\partial x^2} = -2 - 6x \tag{61}$$

Question 2

$$\boldsymbol{H} = \begin{bmatrix} -2-6x & -2y \\ -2y & 2(1-x) \end{bmatrix} \tag{62}$$

Now let's get the determinant:

$$\boldsymbol{H} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \tag{63}$$

$$det(\boldsymbol{H}) = ad - bc \tag{64}$$

Question 2

For (0,0):

$$\boldsymbol{H} = \begin{bmatrix} -2-6\cdot 0 & -2\cdot 0 \\ -2\cdot 0 & 2(1-0) \end{bmatrix} \tag{65}$$

$$\boldsymbol{H} = \begin{bmatrix} -2 & 0 \\ -0 & 2 \end{bmatrix} \tag{66}$$

$$det(\boldsymbol{H}) = -2\cdot 2 - 0\cdot 0 = -4 < 0 \tag{67}$$

Thus, is (0,0) is not a minimum nor a maximum is a saddle point.

Question 2

For (-2/3, 0)

$$\boldsymbol{H} = \begin{bmatrix} -2-6\cdot\frac{-2}{3} & -2\cdot 0 \\ -2\cdot 0 & 2(1-\frac{-2}{3}) \end{bmatrix} \tag{68}$$

$$\boldsymbol{H} = \begin{bmatrix} 2 & 0 \\ 0 & \frac{10}{3} \end{bmatrix} \tag{69}$$

$$det(\boldsymbol{H}) = 2\cdot\frac{10}{3} - 0\cdot 0 = \frac{20}{3} > 0 \tag{70}$$

Thus, is (-2/3, 0) is a minimum .

Question 2

You can do it like this, which is essentially the same:

$$A = f_{xx}(x_0, y_0) \qquad B = f_{xy}(x_0, y_0) \qquad C = f_{yy}(x_0, y_0)$$

i) If $A < 0$ and $AC - B^2 > 0$, then (x_0, y_0) is a (strict) local maximum point.

ii) If $A > 0$ and $AC - B^2 > 0$, then (x_0, y_0) is a (strict) local minimum point.

iii) If $AC - B^2 < 0$, then (x_0, y_0) is a saddle point.

iv) If $AC - B^2 = 0$, then (x_0, y_0) any of the three.

Question 3, a

3. Maximise:

$$f(x_1, x_2) = -x_1^2 - 4x_2 + 10 \tag{71}$$

subject to the constraint $x_1 + x_2 = m$

a) by substitution of the constraint

Question 3, a

We can rearrange the constraint so we get x_2 as a function of m and x_1:

$$x_1 + x_2 = m \tag{72}$$

$$x_2 = m - x_1 \tag{73}$$

Then, we can replace x_2 as a function of x_1 and m into the objective function. We then can derive this function w.r.t to x_1

$$f(x_1, x_2) = -x_1^2 - 4x_2 + 10 \tag{74}$$

$$f(x_1) = -x_1^2 - 4(m - x_1) + 10 \tag{75}$$

Question 3, a

We can rearrange the constraint so we get x_1 as a function of m and x_2:

$$f(x_1) = -x_1^2 - 4(m - x_1) + 10 \tag{76}$$

$$f(x_1) = -x_1^2 - 4m - 4x_1 + 10 \tag{77}$$

We now derive w.r.t to x_1

$$\frac{\partial f(x_1)}{\partial x_1} = \frac{\partial - x_1^2}{\partial x_1} - \frac{\partial 4m}{\partial x_1} + \frac{\partial 4x_1}{\partial x_1} + \frac{\partial 10}{\partial x_1} = 0 \tag{78}$$

$$\frac{\partial f(x_1)}{\partial x_1} = -2 \cdot x_1^{2-1} - 0 + 4 + 0 = 0 \tag{79}$$

Question 3, a

$$\frac{\partial f(x_1)}{\partial x_1} = -2 \cdot x_1^{2-1} - 0 + 4 + 0 = 0 \tag{80}$$

$$\frac{\partial f(x_1)}{\partial x_1} = -2x_1 + 4 = 0 \tag{81}$$

$$\frac{\partial f(x_1)}{\partial x_1} = 2x_1 = 4 \tag{82}$$

$$x_1 = 2 \tag{83}$$

Question 3, a

Now that we have $x_1 = 2$, we can plug it into the constraint function and get x_2

$$x_1 + x_2 = m \tag{84}$$

$$2 + x_2 = m \tag{85}$$

$$x_2 = m - 2 \tag{86}$$

Question 3, b

b) by Lagrange's method

We have our objective function and our budget constraint:

$$f(x_1, x_2) = -x_1^2 - 4x_2 + 10 \tag{87}$$

subject to the constraint $x_1 + x_2 = m$

We set up the Lagrangian:

$$\mathcal{L}(x_1, x_2, \lambda) = -x_1^2 - 4x_2 + 10 - \lambda(x_1 + x_2 - m) \tag{88}$$

or (it's the same):

$$\mathcal{L}(x_1, x_2, \lambda) = -x_1^2 - 4x_2 + 10 + \lambda(m - x_1 - x_2) \tag{89}$$

Question 3, b

$$\mathcal{L}(x_1, x_2, \lambda) = -x_1^2 - 4x_2 + 10 - \lambda(x_1 + x_2 - m) \tag{90}$$

We then proceed to obtain the FOC w.r.t x_1, x_2, λ:

$$\frac{\partial \mathcal{L}(x_1, x_2, \lambda)}{\partial x_1} = -2x_1 - \lambda = 0 \tag{91}$$

$$\frac{\partial \mathcal{L}(x_1, x_2, \lambda)}{\partial x_2} = -4 - \lambda = 0 \tag{92}$$

$$\frac{\partial \mathcal{L}(x_1, x_2, \lambda)}{\partial \lambda} = -x_1 - x_2 + m = 0 \tag{93}$$

Question 3, b

We can easily get λ from the FOC w.r.t to x_2

$$-4 - \lambda = 0 \longrightarrow -4 = \lambda \tag{94}$$

Now that we can replace $\lambda = -4$ into the first FOC:

$$-2x_1 - \lambda = 0 = 0 \longrightarrow -2x_1 + 4 = 0 \tag{95}$$

$$-2x_1 = -4 \longrightarrow x_1 = 2 \tag{96}$$

Now we can take $x_1 = 2$ and plug it into the third FOC:

$$-x_1 - x_2 + m = 0 \longrightarrow -2 - x_2 + m = 0 \longrightarrow -x_2 = -m + 2 \tag{97}$$

$$x_2 = m - 2 \tag{98}$$

Question 4, a

Suppose that a firm, producing output Y form labour L and capital K according to the technology:

$$Y = (K^{0.5} + L^{0.5})^2 \tag{99}$$

wishes to minimise its costs given that is has to produce the amount $\overline{Y}$ of output. The wage rate is w and the rental rate of capital is r, and so its cost is

$$wL + rK \tag{100}$$

a) Derive the first order conditions using the Lagrange's Method

Question 4, a

Let's set up the cost minimisation problem first:

$$\min wL + rK \tag{101}$$

s.t $\overline{Y} = (K^{0.5} + L^{0.5})^2$

How do we set up the Lagrangian in this case?

Question 4, a

$$\mathcal{L}(L, K, \lambda) = wL + rK + \lambda(\overline{Y} - (K^{0.5} + L^{0.5})^2) \qquad (102)$$

We then proceed to obtain the FOC w.r.t L, K and λ

Question 4, a

Let's get the FOC w.r.t L, you can do K and λ on your own:

First, let's remember the chain rule:

$$\frac{d}{dx}\left[f\left(u\right)\right]=\frac{d}{du}\left[f\left(u\right)\right]\frac{du}{dx} \tag{103}$$

$$\mathcal{L}(L,K,\lambda)=wL+rK+\lambda(\overline{Y}-(K^{0.5}+L^{0.5})^2)=0 \tag{104}$$

$$\frac{\partial\mathcal{L}(L,K,\lambda)}{\partial L}=\frac{\partial wL}{\partial L}+\frac{\partial rK}{\partial L}+\frac{\partial\lambda(\overline{Y}-(K^{0.5}+L^{0.5})^2)}{\partial L}=0 \tag{105}$$

$$\frac{\partial(L,K,\lambda)}{\partial L}=w+0-\frac{\partial\lambda(K^{0.5}+L^{0.5})^2}{\partial L}=0 \tag{106}$$

Question 4, a

$$\frac{\partial(L,K,\lambda)}{\partial L} = w - \frac{\partial \lambda (K^{0.5} + L^{0.5})^2}{\partial L} = 0 \quad (107)$$

$$\frac{\partial(L,K,\lambda)}{\partial L} = w - 2\lambda (K^{0.5} + L^{0.5})^{2-1} \cdot \frac{\partial L^{0.5}}{\partial L} = 0 \quad (108)$$

$$\frac{\partial(L,K,\lambda)}{\partial L} = w - \cancel{2}\lambda (K^{0.5} + L^{0.5}) \cdot \frac{1}{\cancel{2}} L^{0.5-1} = 0 \quad (109)$$

$$\frac{\partial(L,K,\lambda)}{\partial L} = w - \cancel{2}\lambda (K^{0.5} + L^{0.5}) \cdot \frac{1}{\cancel{2}} L^{-0.5} = 0 \quad (110)$$

Question 4, a

$$w - \lambda L^{-0.5}(K^{0.5} + L^{0.5}) = 0 \tag{111}$$

If you do the same for the other two FCOs, you get the following:

$$r - \lambda K^{-0.5}(K^{0.5} + L^{0.5}) = 0 \tag{112}$$

$$(K^{0.5} + L^{0.5})^2 - \overline{Y} = 0 \tag{113}$$

Question 4, b

b) Show that the firm's demand for labour and capital are given by:

$$L^*(w, r, \overline{Y}) = \frac{\overline{Y}}{(1 + \frac{w}{r})^2} \tag{114}$$

$$K^*(w, r, \overline{Y}) = \frac{\overline{Y}}{(1 + \frac{r}{w})^2} \tag{115}$$

Question 4, b

How do we obtain the demand for each input?

Question 4, b

If you compute the three FOCs, you will get the three equations below, and you need to solve for this system of equations to find the demand for each input.

$$w - \lambda L^{-0.5}(K^{0.5} + L^{0.5}) = 0 \tag{116}$$

$$r - \lambda K^{-0.5}(K^{0.5} + L^{0.5}) = 0 \tag{117}$$

$$(K^{0.5} + L^{0.5})^2 - \overline{Y} = 0 \tag{118}$$

Question 4, b

Alternatively, a profit maximising firm will demand from each input the marginal product of that input equals to the price of that input.

$$\begin{aligned} w &= p \times MP_L \\ r &= p \times MP_K \end{aligned} \tag{119}$$

$$\frac{w}{r} = \frac{MP_L}{MP_K} \tag{120}$$

$$\frac{w}{r} = \frac{\lambda L^{-0.5}(K^{0.5} + L^{0.5})}{\lambda K^{-0.5}(K^{0.5} + L^{0.5})} \tag{121}$$

$$\frac{w}{r} = \frac{\cancel{\lambda} L^{-0.5}\cancel{(K^{0.5} + L^{0.5})}}{\cancel{\lambda} K^{-0.5}\cancel{(K^{0.5} + L^{0.5})}} \tag{122}$$

Question 4, b

Alternatively, a profit maximising firm will demand from each input the marginal product of that input equals to the price of that input.

$$\frac{w}{r} = \frac{\cancel{X} L^{-0.5} \cancel{(K^{0.5} + L^{0.5})}}{\cancel{X} K^{-0.5} \cancel{(K^{0.5} + L^{0.5})}} \tag{123}$$

$$\frac{w}{r} = \frac{L^{-0.5}}{K^{-0.5}} \longrightarrow \frac{\frac{1}{L^{0.5}}}{\frac{1}{K^{0.5}}} \longrightarrow \frac{K^{0.5}}{L^{0.5}} \tag{124}$$

$$\frac{w}{r} = \frac{K^{0.5}}{L^{0.5}} \tag{125}$$

$$\frac{wL^{0.5}}{r} = K^{0.5} \tag{126}$$

Question 4, b

We can then replace $K^{0.5}$ into the FOC w.r.t λ

$$(K^{0.5} + L^{0.5})^2 - \overline{Y} = 0 \tag{127}$$

$$\frac{wL^{0.5}}{r} = K^{0.5} \tag{128}$$

$$(\frac{wL^{0.5}}{r} + L^{0.5})^2 - \overline{Y} = 0 \tag{129}$$

$$(\frac{wL^{\frac{1}{2}}}{r} + L^{1/2})^2 = \overline{Y} \tag{130}$$

$$(L^{\frac{1}{2}})^2(\frac{w}{r} + 1)^2 = \overline{Y} \tag{131}$$

Question 4, b

$$(L^{\frac{1}{2}})^2(\frac{w}{r}+1)^2 = \overline{Y} \tag{132}$$

$$(L)^{\frac{1}{\cancel{2}}\cdot\cancel{2}}(\frac{w}{r}+1)^2 = \overline{Y} \tag{133}$$

$$L^*(w,r,\overline{Y}) = \frac{\overline{Y}}{(\frac{w}{r}+1)^2} \tag{134}$$

Try to get on your own the demand for capital K^*.

Question 5

5. Suppose that a firm using capital (K) and labour (L) has the following production function:

$$F(K, L) = AK^{\alpha}L^{\beta} \tag{135}$$

with A, $\alpha, \beta > 0$

a. Find the marginal product of labour and the marginal product of capital. Show that these are positive and decreasing only if $0 < \alpha < 1$ and and $0 < \beta < 1$.

Question 5, a)

How do we get the marginal product of each input?

Question 5, a)

$$F(K,L) = AK^{\alpha}L^{\beta} \tag{136}$$

Let's get the MP of Labour

$$MP_L = \frac{\partial F(K,L)}{\partial L} = \frac{\partial AK^{\alpha}L^{\beta}}{\partial L} \tag{137}$$

$$MP_L = \frac{\partial F(K,L)}{\partial L} = A\beta K^{\alpha}L^{\beta-1} \tag{138}$$

$$MP_L = \frac{A\beta K^{\alpha}}{L^{1-\beta}} \tag{139}$$

Try to get on your own the MP_K, but this is the answer:

$$MP_K = \frac{A\alpha L^{\beta}}{K^{1-\alpha}} \tag{140}$$

Question 5, a)

Are they decreasing or increasing? Let's see for the MP_L. What happens if $\beta > 1$ and $\alpha > 1$

$$MP_L = \frac{A\beta K^{\alpha} \uparrow}{L^{(1-\beta)<0}} \tag{141}$$

$$MP_L = \frac{A\beta K^{\alpha} \uparrow}{\frac{1}{L^{(1-\beta)>0}}} \tag{142}$$

$$MP_L = (A\beta K^{\alpha}) \uparrow \cdot (L^{(1-\beta)>0}) \uparrow >> 0 \tag{143}$$

Question 5, a)

If $0 < \alpha < 1$ and and $0 < \beta < 1$

$$MP_L = \frac{A \overbrace{\beta}^{\text{Decrease}} K^{\alpha \text{Decreasing}}}{\underbrace{\overset{\text{Decreasing}}{L^{1-\beta}}}} \tag{144}$$

Let put plug some numbers, let's say $\alpha = 0.5$ and $\beta = 0.5$

$$MP_L = \frac{A0.5K^{0.5}}{L^{1-0.5}} \tag{145}$$

$$MP_L = \frac{A0.5\sqrt{K}}{\sqrt{L}} \tag{146}$$

Quadratic functions are decreasing, plus we are multiplying the numerator by 0.5.

Question 5, a)

Then, the asked us to show that is **positive** if $0 < \alpha < 1$ and and $0 < \beta < 1$

Let's remember that a function is decreasing/increasing if:

- If $f''(x) < 0$ for x, then $f(x)$ is positive
- If $f''(x) > 0$ for x, then $f(x)$ is negative

Let's get the second partial derivatives.

Question 5, a)

$$MP_L = \frac{A\beta K^{\alpha}}{L^{1-\beta}} \tag{147}$$

$$\frac{\partial MP_L}{\partial L} = A\beta K^{\alpha} L^{-(1-\beta)} \tag{148}$$

$$\frac{\partial MP_L}{\partial L} = -(1-\beta)A\beta K^{\alpha} L^{-(1-\beta)-1} \tag{149}$$

$$\frac{\partial MP_L}{\partial L} = -(1-\beta)A\beta K^{\alpha} L^{-\beta-2} \tag{150}$$

$$\frac{\partial MP_L}{\partial L} = \frac{-(1-\beta)A\beta K^{\alpha}}{L^{2-\beta}} \tag{151}$$

Question 5, a)

Let's see if it's positive if $\alpha = 0.5$ and $\beta = 0.5$

$$\frac{\partial MP_L}{\partial L} = \frac{-\overbrace{(1-\beta)}^{\text{Positive}}\ \overbrace{A}^{\text{Positive}}\ \beta\ \overbrace{K^{\alpha}}^{\text{Positive}}}{\underbrace{L^{2-\beta}}_{\text{Positive}}} \tag{152}$$

$$\frac{\partial MP_L}{\partial L} < 0 \tag{153}$$

Then, given that our second partial derivative is negative, we can say that is positive.

Question 5, a)

Try to do own your own the second partial derivative w.r.t K, but here is the answer:

$$\frac{\partial MP_K}{\partial K} = \frac{-(1-\alpha)A\alpha L^{\beta}}{L^{2-\alpha}} \tag{154}$$

Question 5, b)

b) What is the degree of homogeneity of this production function? (Recall that a function $f(x, y)$ is homogeneous of degree t if $f(kx, ky) = k^t f(x, y)$ for any $k > 0$.

How we identify the degree of homogeneity in a function?

Question 5, b)

You basically have to find the value of t.

$$f(kx, ky) = k^t f(x, y) \tag{155}$$

$$F(K, L) = AK^\alpha L^\beta \tag{156}$$

Let's multiply each factor by k.

$$F(kK, kL) = A(kK)^\alpha (kL)^\beta \tag{157}$$

$$F(kK, kL) = k^\alpha K^\alpha k^\beta L^\beta \tag{158}$$

$$F(kK, kL) = k^{\alpha+\beta} AK^\alpha L^\beta, \tag{159}$$

Question 5, b)

$$\text{But, we know that:} F(K, L) = AK^{\alpha}L^{\beta} \tag{160}$$

$$F(kK, kL) = k^{\alpha+\beta}F(K, L) \tag{161}$$

Then:

$$f(kx, ky) = k^{t}f(x, y) \tag{162}$$

$$k^{t} = k^{\alpha+\beta}F(K, L) \tag{163}$$

$$t = \alpha + \beta \tag{164}$$

Hence, this production function is homogeneous of degree α and β.

- If $t > 1$ the function displays increasing returns to scale
- If $t < 1$ the function display decreasing returns to scale

Question 2 - extension

By request of some students here is the solution solving by substitution

We have to meet these two equations and we need to find x and y that meet both equalities:

$$2y(1-x) = 0 \tag{165}$$

$$-y^2 - 2x - 3x^2 = 0 \tag{166}$$

Let's find y as a function of x:

$$-y^2 = 2x + 3x^2 \tag{167}$$

$$y^2 = -2x - 3x^2 \longrightarrow y = \sqrt{-2x - 3x^2} \tag{168}$$

Question 2 - extension

We plug in y as a function of x into equation 165.

$$2(\sqrt{-2x-3x^2})(1-x) = 0 \text{ We divide in both side by } 1/2 \tag{169}$$

$$\sqrt{-2x-3x^2}(1-x) = 0 \tag{170}$$

You could then, solve for x when $\sqrt{-2x-3x^2} = 0$.

$$\sqrt{-2x-3x^2} = 0 \longrightarrow -2x-3x^2 = 0 \longrightarrow -2x = 3x^2 \tag{171}$$

$$-2x = 3x^2 \longrightarrow x = \frac{-2}{3} \tag{172}$$

Question 2 - extension

What about $(1 - x) = 0$. Then. $x = 1$. But we saw that if $x = 1$ is not a possible solution.

Getting x by multiplying all the elements inside of each parentheses $((\sqrt{-2x - 3x^2})(1 - x) = 0)$ yields a cubic equation, which is somewhat hard to solve.

Microeconomics (5SSPP217)

Seminar 1

Felipe Torres
felipe.torres@kcl.ac.uk

June 4, 2025

Contact & Office Hours

- My email: **felipe.torres@kcl.ac.uk**
- Office hours: Every Wednesday from 13:00 until 14:00. Next week: Bush House (NE) WING. 7.22.
- From Wednesday 20 October. BH (SE)2.06
- I will send you the slides once the solution sheet is made available on KEATS.

Refresh your memory

Assumptions about preferences:

- Complete: Any given x-bundle and y-bundle, we assume that $(x_1, x_2) \succeq (y_1, y_2)$ or $(y_1, y_2) \succeq (x_1, x_2)$
- Reflexive: Any bundle x is at least as good as itself: $(x_1, x_2) \succeq (x_1, x_2)$
- Transitive: If $(x_1, x_2) \succeq (y_1, y_2)$ and $(y_1, y_2) \succeq (z_1, z_2)$ then $(x_1, x_2) \succeq (z_1, z_2)$

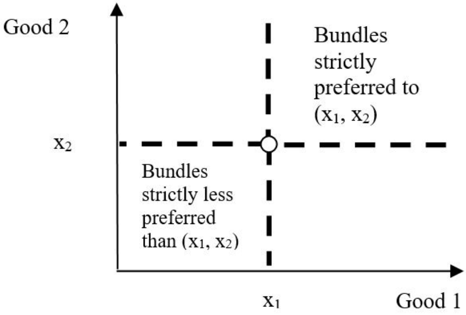
Question 1, a

1. Wen's preferences over good 1 and 2 are as follows
$(y_1, y_2) \precsim (x_1, x_2)$ if and only if $y_1 \leq x_1$ **and** $y_2 \leq x_2$

a) For any one allocation (x_1, x_2), draw in the space of Wen's consumption of good 1 and good 2 the set of bundles Wen strictly prefers to (x_1, x_2), and the set of bundles which Wen prefers strictly less than (x_1, x_2)

Question 1, a

1.a) Given the monotonicity assumption, we know that if we increase the amount of any of the goods of a bundle, we will strictly prefer the new bundle.



Question 1, b

1. Wen's preferences over good 1 and 2 are as follows $(y_1, y_2) \precsim (x_1, x_2)$ if ad only if $y_1 \leq x_1$ and $y_2 \leq x_2$?

b) Are Wen's preferences complete, monotonic, and transitive?

Question 1, b

1.b) **No!** Why?

- We cannot order bundles such that $y_1 \leq x_1$ and $x_2 \leq y_2$

Question 1, a

1.a) Are Wen's preferences monotone? **Yes**, If you increase the amount of any good, the new bundle is better.

Let's choose a bundle that is $Z = (z_1, z_2)$ such that $x_1 <= z_1$ and $x_2 <= z_2$. Then, based on Wen's preference relation, he will choose bundle Z, which at least the same or more than bundle X Therefore his preference are monotonic.

$$(x_1, x_2) \precsim (z_1, z_2) \tag{1}$$

We could have stated for example, that $x_1 = z_1$ and $x_2 <= z_2$. In this case, Wen's preferences are again monotonic.

$$(x_1, x_2) \precsim (z_1, z_2) \tag{2}$$

Question 1, a

1.a) Are Wen's preferences transitive? **Yes**

Proof:

We know that:

- $y_1 \leq x_1$ and $y_2 \leq x_2 \longrightarrow (y_1, y_2) \precsim (x_1, x_2)$ and
- $z_1 \leq y_1$ and $z_2 \leq y_2 \longrightarrow (z_1, z_2) \precsim (y_1, y_2)$ then
- $z_1 \leq x_1$ and $z_2 \leq x_2 \longrightarrow (z_1, z_2) \precsim (x_1, x_2)$
- $(z_1, z_2) \precsim (y_1, y_2) \precsim (x_1, x_2)$

Question 2

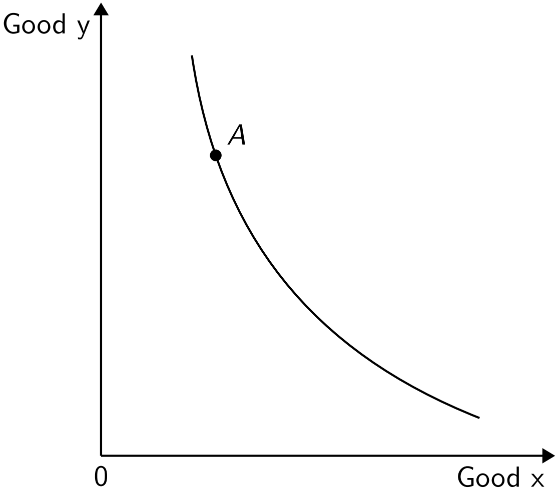
2) Angela's preferences over bundles of two goods satisfy completeness, transitivity, monotonicity, and convexity. Consider two bundles: A = (6, 12) and B = (18; 6). Her preferences are that such $X \backsim Y$. Can you determine whether Angela would prefer bundle C=(14; 8) to A and B with the given information?

Question 2

Let's remember what it means to have **convex** preferences.

- Indifference curves have a *negative* slope

Question 2



Question 2

Let's remember what it means to have **convex** preferences.

- Indifference curves have a *negative* slope
- Any weighted average of two indifferent bundles is at least *weakly* preferred to the extreme bundles
- Formally: Suppose that we have bundle $x, y, z \in X$, where $z \succeq y$ and $y \succeq z$ for every $t \in [0, 1]$, then

$$\underbrace{(t \cdot x_1 + (1-t) \cdot y_1, t \cdot x_2 + (1-t) \cdot y_2)}_{z} \succsim (x_1, x_2) \tag{3}$$

Question 2

From the question we know that:

- We have two bundles A = (6, 12) and B = (18; 6) and they are in the same indifference curve as $X \backsim Y$
- So the question is how can we re-weight these two bundles to see if bundle C is weakly or strictly prefer to A and/or B?

$$(t \cdot 6 + (1 - t) \cdot 18; t \cdot 12 + (1 - t) \cdot 6) = (14, 8)^1 \tag{4}$$

$$6t + 18 - 18t = 14 \tag{5}$$

$$-12t = 14 - 18 \tag{6}$$

$$-12t = -4 \tag{7}$$

$$t = \frac{1}{3} \tag{8}$$

Question 2

We replace $t = \frac{1}{3}$ into the equation:

$$(\frac{1}{3} \cdot 6 + (1 - \frac{1}{3}) \cdot 18; \frac{1}{3} \cdot 12 + (1 - \frac{1}{3}) \cdot 6) = (14, 8) \tag{9}$$

$$(\frac{1}{3} \cdot 6 + (\frac{2}{3}) \cdot 18; \frac{1}{3} \cdot 12 + (\frac{2}{3}) \cdot 6) = (14, 8) \tag{10}$$

$$(2 + 12; 4 + 4) \tag{11}$$

$$(14; 8) \tag{12}$$

Therefore, C is a combination of A and B, and thus, at least weakly preferred.

Question 2

But more generally, we know that whenever we have two bundles (A and B) that are in the same indifference curve. Then, any bundle Z, that is a combination of such bundles, then $A \preceq Z$ and $B \preceq Z$

$$C = \frac{1}{3} \cdot A + \frac{2}{3} \cdot B \tag{13}$$

Question 3

3) Dr Pages' utility function is $U(s, t) = s^{1/2} \cdot t^{1/3}$, where s is the amount of scones he consumes and t the amount of cups of tea he consumes. He has a budget of B pounds and faces prices of p_s for each scone he buys and a price of 1 for each cup of tea he buys. **Use the Lagrange method** to determine the optimal quantities of scones and tea for Dr Pages (i.e. his demand for each good as functions of p_s and B).

Question 3

3) First, what is Dr Pages' budget constraint?

- quantity of scones: s
- price scones: p_s
- quantity tea: t
- price tea: 1
- Budget: B

Thus, we can write the budget constraint as:

$$t + p_s \cdot s = B \tag{14}$$

Question 3

3) Second, we know they are asking us to maximise Dr Pages' utility function: $U(s,t) = s^{1/2} \cdot t^{1/3}$. We then can pose the utility maximisation problem as a constrained maximisation problem:

$$\begin{aligned} \max \quad & s^{1/2} \cdot t^{1/3} \\ \text{s.t.} \quad & t + p_s \cdot s = B \end{aligned} \tag{15}$$

Question 3

3) How do we solve for this utility maximisation problem in a systematic way, using calculus conditions for maximisation?

Answer: Using the Lagragian Method or Lagrange Multipliers, which is defining an auxiliary function known as the *Lagragian*.

$$L(s;t;\lambda) = s^{1/2} \cdot t^{1/3} - \lambda \cdot (t + p_s \cdot s - B) \tag{16}$$

Question 3

3) Third, the Lagrange's theorem says that an optimal choice (x_1^*, x_2^*) must satisfy the three first-order conditions:

Let's start by obtaining the first-order condition with respect to s:

$$\frac{\partial L}{\partial s} = \frac{\partial s^{\frac{1}{2}} \cdot t^{\frac{1}{3}}}{\partial s} - \frac{\partial \lambda \cdot (t + p_s \cdot s - B)}{\partial s} = 0 \tag{17}$$

$$\frac{\partial L}{\partial s} = \frac{1}{2} s^{\frac{1}{2}-1} \cdot t^{\frac{1}{3}} - \lambda \cdot p_s = 0 \tag{18}$$

$$\frac{\partial L}{\partial s} = \frac{1}{2} s^{\frac{-1}{2}} \cdot t^{\frac{1}{3}} - \lambda \cdot p_s = 0 \tag{19}$$

Question 3

3) You can obtain the FOCs with respect to t and λ::

$$\frac{\partial L}{\partial t} = \frac{1}{3} s^{\frac{1}{2}} t^{\frac{-2}{3}} - \lambda = 0 \tag{20}$$

$$\frac{\partial L}{\partial \lambda} = -t - p_s \cdot s + B = 0 \tag{21}$$

From the second FOC, λ is equal to:

$$\lambda = \frac{1}{3} s^{\frac{1}{2}} t^{\frac{-2}{3}} \tag{22}$$

Question 3

3) Let's solve first s as function of p_s and B. In the following step I plug in λ into the FOC w.r.t to s.

$$\frac{1}{2}s^{\frac{-1}{2}} \cdot t^{\frac{1}{3}} - \lambda \cdot p_s = 0 \tag{23}$$

$$s^{\frac{-1}{2}} = \frac{2\lambda \cdot p_s}{t^{\frac{1}{3}}} \tag{24}$$

Let's plug in λ

$$\frac{1}{3}s^{\frac{1}{2}} t^{\frac{-2}{3}} = \lambda \text{ We replace by } \lambda \tag{25}$$

$$s^{\frac{-1}{2}} = \frac{\frac{2s^{\frac{1}{2}} t^{\frac{-2}{3}} p_s}{3}}{t^{\frac{1}{3}}} \longrightarrow \frac{2s^{\frac{1}{2}} t^{\frac{-2}{3}} p_s}{3} \cdot \frac{1}{t^{1/3}} \tag{26}$$

$$s^{\frac{-1}{2}} = \frac{2s^{\frac{1}{2}} t^{\frac{-2}{3} - \frac{1}{3}} p_s}{3} \tag{27}$$

Question 3

3) Let's solve first s as function of p_s and B

$$s^{\frac{-1}{2}} = \frac{2}{3} s^{\frac{1}{2}} t^{-1} p_s \tag{28}$$

$$s^{\frac{-1}{2}} = \frac{2s^{\frac{1}{2}} p_s}{3t} \tag{29}$$

$$\frac{s^{\frac{-1}{2}}}{s^{\frac{1}{2}}} = \frac{2p_s}{3t} \tag{30}$$

$$s^{\frac{-1}{2} - \frac{1}{2}} = \frac{2p_s}{3t} \tag{31}$$

$$s^{-1} = \frac{2p_s}{3t} \tag{32}$$

Question 3

3) Let's solve first s and then get t.

$$s = \frac{3t}{2p_s} \tag{33}$$

We also know from the FOC with respect to λ that t is equal to:

$$t = B - p_s s \tag{34}$$

$$t = B - p_s s \cdot \frac{3t}{2p_s} \tag{35}$$

$$t = B - \cancel{p_s} s \cdot \frac{3t}{2\cancel{p_s}} \tag{36}$$

$$t = B - \frac{3t}{2} \tag{37}$$

Question 3

3) Let's solve first s and then get t t

$$t = B - \frac{3t}{2} \tag{38}$$

$$t + \frac{3t}{2} = B \tag{39}$$

$$\frac{2t + 3t}{2} = B \tag{40}$$

$$\frac{5t}{2} = B \tag{41}$$

$$t^* = \frac{2B}{5} \tag{42}$$

Question 3

3) Now that we know t, let's solve for s:

$$s = \frac{3t}{2p_s} \text{ and } t^* = \frac{2B}{5} \tag{43}$$

$$s = \frac{\frac{3 \cdot 2B}{5}}{2p_s} \tag{44}$$

$$s = \frac{\frac{6B}{5}}{2p_s} \tag{45}$$

$$s = \frac{6B}{5} : 2p_s \longrightarrow \frac{6B}{5} \cdot \frac{1}{2p_s} \longrightarrow \frac{3B}{5p_s} \tag{46}$$

$$s^* = \frac{3B}{5p_s} \tag{47}$$

Question 4, a

4) Mr Richard's preferences can be represented with the utility function $U(x, y) = 2x + 5y$, where x and y are the amounts of the only two goods he cares about. His budget is £150

a) Suppose the prices of good and x and y are $p_x = 4$ and $p_y = 15$ respectively. Draw Mr Richards' budget constraint and a few indifference curves. What is Mr Richards' optimal consumption bundle? What is the level of "happiness" (i.e. utility) Mr Richards attains when consuming this bundle?

Question 4, a

a) Draw Mr Richards' budget constraint and a few indifference curves.
We know that Mr Richards' utility function is the following:

$$U(x, y) = 2x + 5y \tag{48}$$

We can replace U(x,y) to k, where k can be different levels of utility

$$k = 2x + 5y \tag{49}$$

We can rewrite this function as y as a function of k and 2x:

$$y = \frac{k}{5} - \frac{2x}{5} \tag{50}$$

Question 4, a

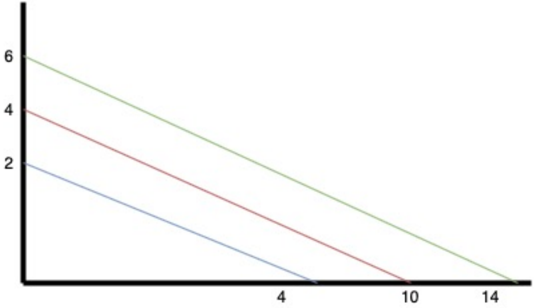
a) Draw Mr Richards' budget constraint and a few indifference curves.

y	k	x	2x/5
2	10	0	0
0	10	5	2
4	20	0	0
0	20	10	4
6	30	0	0
0	30	15	6

Table: Inferences curves question 4

Question 4, a

a) Draw Mr Richards' budget constraint and a few indifference curves.



Question 4, a

a) What is Mr Richards' optimal consumption bundle? What is the level of "happiness" (i.e. utility) Mr Richards attains when consuming this bundle?

- Quantity of x
- Price of x $p_x = 4$
- Quantity of y
- Price of y $p_y = 15$

$$4x + 15y = 150 \tag{51}$$

Question 4, a

We can rearrange y as function of x

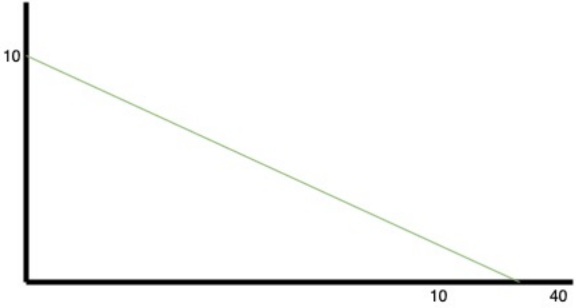
$$4x + 15y = 150 \tag{52}$$

$$15y = 150 - 4x \tag{53}$$

$$y = 10 - \frac{4x}{15} \tag{54}$$

Question 4, a

We can draw the budget constraint:



Question 4, a

We plug in y into the utility function $U(x, y)$

$$U(x, y) = 2x + 5y \tag{55}$$

$$U(x) = 2x + 5(10 - \frac{4x}{15}) \tag{56}$$

$$U(x) = 2x + 50 - 5\frac{4x}{15} \tag{57}$$

$$U(x) = 2x + 50 - 5\frac{4x}{3} \tag{58}$$

$$U(x) = \frac{6x - 4x}{3} + 50 \tag{59}$$

$$U(x) = \frac{2x}{3} + 50 \tag{60}$$

Question 4, a

What can we tell about this equation?

$$U(x) = \frac{2x}{3} + 50 \tag{61}$$

That is increasing in x, meaning the more we consume on x, the greater our utility. Consequently, all income is spent on x; therefore, we reached a corner solution.

Just to make sure, if you write the utility function as function of y, you will find that the utility function is decreasing in y.

$$U(y) = 75 - 2.5y \tag{62}$$

Question 4, a

Alternatively, you could use the Marginal Rate of Substitution against prices to determine how Mr Richards will spend his income:

$$\frac{\partial U(x,y)}{\partial x} = 2 \tag{63}$$

$$\frac{\partial U(x,y)}{\partial y} = 5 \tag{64}$$

Thus, our MRS is equal to:

$$MRS = 2/5 \text{ and we know that } \frac{p_x}{p_y} = 4/15 \tag{65}$$

$$\frac{2}{5} > \frac{4}{15} \tag{66}$$

You want to sell of more good 2 to get more of good 1. The amount of good 2 you must sell is lower than the one you are willing to give up.

Question 4, a

What is Mr Richards' optimal consumption bundle?
He spends all of his income in good x, therefore

$$x = \frac{150}{4} = 37.5 \tag{67}$$

What is the level of "happiness" Mr Richards attains when consuming this bundle? (x;y) = (37,5; 0)

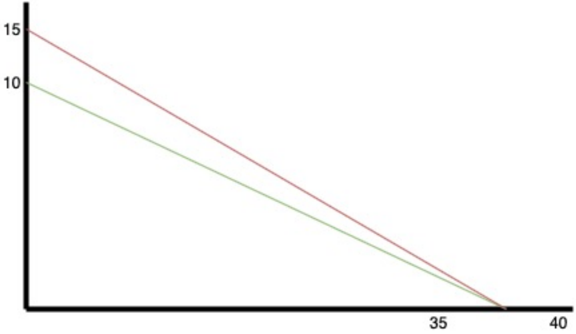
$$U(x, y) = 2x + 5y \tag{68}$$

$$U(37.5, 0) = 2 \cdot 37.5 + 5 \cdot 0 \tag{69}$$

$$U(37.5, 0) = 75 \tag{70}$$

Question 4, a

Indifference curve and Budget constraint



Question 4, b

b) Suppose now that Mr Richards can join a buyer's club offering a discount in the price of good y from 15 to 10. How would his budget constraint change if he join the buyer's club?

Question 4, b

b) This means that we have a new price of y, $p_y = 10$, thus Mr Richards' budget constraint is as follows:

$$150 = 4x + 10y \tag{71}$$

$$y = 15 - \frac{4x}{10} \tag{72}$$

We substitute y in Mr Richards' utility function:

$$U(x) = 2x + 5(15 - \frac{4x}{10}) \tag{73}$$

$$U(x) = 2x + 75 - \frac{20x}{10} \text{ which is equal to } 75 \tag{74}$$

Question 4, b

b) This means that our utility is constant: any bundle on the budget line gives the same utility $(MRS = \frac{p_x}{p_y})$.

Since the level of utility is the same as before, the consumer is not willing to pay for joining the club.

Question 5, a

5. You are the major of a little town. Your budget is £38k and you must decide how much to allocate to fund sports facilities (good x) and cultural activities (good y). Each sport activity costs £2k and each cultural activity £1k. The preferences of your citizens over sports and cultural activities can be represented by the utility function $U(x, y) = 2\ln(x) + y$

a) Draw the budget constraint and a few indifference curves (make sure to derive the mathematical expression for the curves before drawing them).

Question 5, a

5.a) Draw the budget constraint:

We know that:

- Sport facilities x
- Cultural activities y
- Price sport facilities $p_x = 2$
- Price cultural activities $p_y = 1$
- Total budget £$38k$

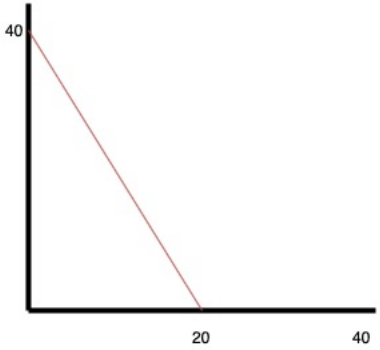
$$38 = 2x + y \tag{75}$$

To draw we can rearrange the equation so y is a function of x

$$y = 38 - 2x \tag{76}$$

Question 5, a

5.a) Draw the budget constraint:



Question 5, a

5.a) Draw a few indifference curves
We can rewrite our utility function as a fixed amount k:

$$U(x,y) = 2\ln(x) - y \tag{77}$$

We can take the first derivative U(x,y) w.r.t x and y

$$\frac{\partial U(x,y)}{\partial x} = \frac{2}{x} \tag{78}$$

$$\frac{\partial U(x,y)}{\partial y} = -1 \tag{79}$$

$$MRS = \frac{\frac{\partial U}{\partial x}}{\frac{\partial U}{\partial y}} = \frac{\frac{2}{x}}{-1} = \frac{-2}{x} \tag{80}$$

Question 5, a

$$y = k - 2\ln(x) \tag{81}$$

$$\frac{\partial y}{\partial x} = -\frac{2}{x} < 0 \tag{82}$$

This means that the indifference curve has diminishing MRS, which translates into a flatter indifference curve as x increases.

Question 5, a

5.a) Draw a few indifference curves
First, we will rewrite the utility function as function of y, and then try different levels of utility:

$$U(x, y) = 2\ln(x) - y \tag{83}$$

We can rewrite y as a function of U(x,y) and x, where U(x,y) = k

$$y = k - 2\ln(x) \tag{84}$$

Now that we have y as a function x, we can draw the indifference curves more easily.

Question 5, a

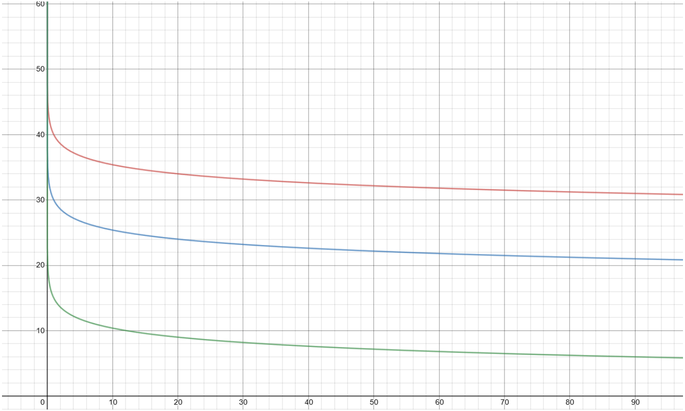
Let's draw k = 36, 30, and 20, and x = 50 and 20

Y	k	x	ln(x)	2ln(x)
28.18	36	3.91	50	7.82
30	36		20	5.99
22	30		50	7.82
24	30		20	5.99
12.17	20		50	7.82
14	20		20	5.99

Table: Indifference curves

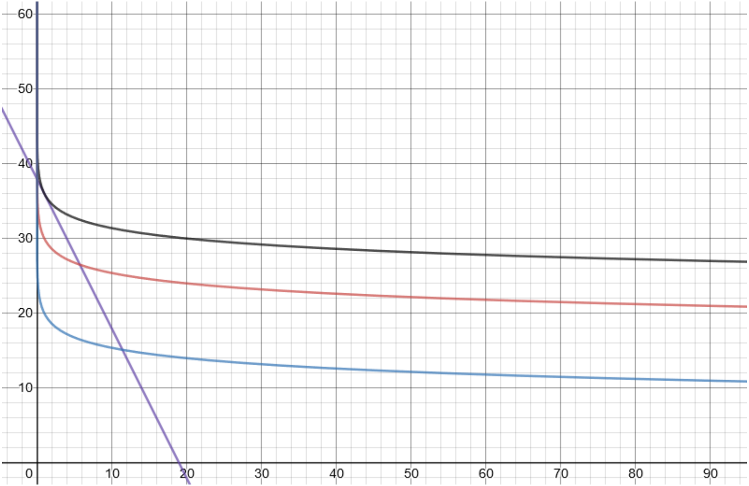
Question 5, a

5.a) This is what we get:



Question 5, a

5.a) Both graphs together:



Question 5, b

5.b) We use the Lagrangian method:

$$\begin{aligned} \max \quad & 2\ln(x) + y \\ \text{s.t.} \quad & 2x + y = 38 \end{aligned} \tag{85}$$

remember that:

$$\frac{\partial ln(x)}{\partial x} = \frac{1}{x} \tag{86}$$

The Lagrangian function is:

$$L(x; y; \lambda) = 2\ln(x) + y - \lambda(2x + y - 38) \tag{87}$$

Question 5, b

The Lagrangian function is:

$$L(x; y; \lambda) = 2\ln(x) + y - \lambda(2x + y - 38) \tag{88}$$

FOCs:

$$\frac{\partial L}{\partial x} = \frac{2}{x} - 2\lambda = 0 \tag{89}$$

$$\frac{\partial L}{\partial y} = 1 - \lambda = 0 \tag{90}$$

$$\frac{\partial L}{\partial \lambda} = 2x + y - 38 = 0 \tag{91}$$

Question 5, b

Solving this system of equations:

$$\lambda = 1 \tag{92}$$

$$\frac{2}{x} - 2\lambda = 0 \tag{93}$$

$$x = \frac{\cancel{2} \cdot 1}{\cancel{2}} \text{ Then, we plug in x = 1 into the 3er FOC} \tag{94}$$

$$2x + y - 38 = 0 \tag{95}$$

$$2 \cdot 1 + y - 38 = 0 \tag{96}$$

$$y^* = 36 \tag{97}$$

Question 5, b

Solving for this system of equations yields optimal choices:

- $x^* = 1$
- $y^* = 36$

Question 6

6. A consumer has perfect complements preferences over goods 1 and 2 represented by the utility function $U(x1, x2) = \min\{x_1 + 2x_2, 2x_1 + x_2\}$. The budget of the consumer is m monetary units. Find the optimal choice of good 1 and good 2 for this consumer.

Question 6

- We are dealing here with Leontieff preferences $U(ax_1, bx_2) = \min\{ax_1, bx_2\}$, thus the optimum condition between two bundles is $ax_1 = bx_2$.
- These types of preferences are (weakly monotonic), thus $p_1x_1 + p_2x_2 = m$ We want to exhaust our budget

Question 6

We have two equations and two unknowns:

- The optimal condition: $x_1 + 2x_2 = 2x_1 + x_2$
- $p_1x_1 + p_2x_2 = m$

Let's work out first the optimal condition:

$$x_1 + 2x_2 = 2x_1 + x_2 \tag{98}$$

$$2x_2 - x_2 = 2x_1 - x_1 \tag{99}$$

$$x_2 = x_1 \tag{100}$$

Question 6

We can know replace x_1 as x_2 into our budget constraint function

$$p_1x_1 + p_2x_2 = m \tag{101}$$

$$p_1x_2 + p_2x_2 = m \tag{102}$$

$$x_2 \cdot (p_1 + p_2) = m \tag{103}$$

$$x_2^* = \frac{m}{p_1 + p_2} \text{ similarly} \tag{104}$$

$$x_1^* = \frac{m}{p_1 + p_2} \tag{105}$$

Question 6

We know that at any optimal choice of a consumer with perfect complement preferences, the two arguments of their utility function must equate. Otherwise they are spending too much on one good or the other.

This means that at any optimal choice $x_1 + 2x_2 = 2x_1 + x_2$ which implies $x_1 = x_2$. This implies case is identical to the one we seen in the lecture with $a = b = 1$ and hence the optimal choices are $x_1^* = x_2 * = \frac{m}{p_1+p_2}$

Question 6

If we want to draw the indifference curves for this utility function, we set an arbitrary utility level k as we did before:
There are two scenarios, when:

$$x_1 + 2x_2 < 2x_1 + x_2 \text{ which is equal to } x_1 < x_2 \tag{106}$$

$$U(x_1, x_2) = \min\{x_1 + 2x_2, 2x_1 + x_2 = x_1 + 2x_2\} \tag{107}$$

We can replace $U(x_1, x_2)$ to k, which is an arbitrary level of utility:

$$k = x_1 + 2x_2 \text{ We rearrange } x_2 \text{ as function of k and } x_1 \tag{108}$$

$$x_2^* = \frac{k - x_1}{2} \tag{109}$$

Question 6

The second scenario is when $x_1 + 2x_2 > 2x_1 + x_2$

$$x_1 + 2x_2 > 2x_1 + x_2 \text{ which is equal to } x_1 > x_2 \tag{110}$$

$$U(x_1, x_2) = \min\{x_1 + 2x_2, 2x_1 + x_2\} = 2x_1 + x_2 \tag{111}$$

We can replace $U(x_1, x_2)$ to k, which is an arbitrary level of utility:

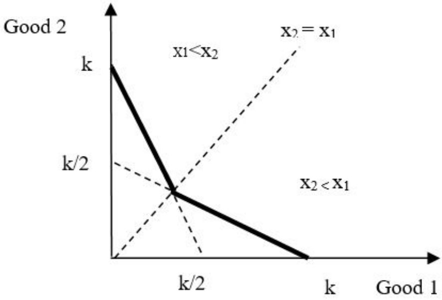
$$k = 2x_1 + x_2 \text{ We rearrange } x_2 \text{ as function of k and } x_1 \tag{112}$$

$$x_2^* = k - 2x_1 \tag{113}$$

Question 6

Therefore, the mathematical expressions for the indifference curve is

$$x_2^* = \begin{cases} k - 2x_1, & \text{if } x_1 < x_2 \\ \frac{k-x_1}{2}, & \text{if } x_1 > x_2 \end{cases} \tag{114}$$



Question 6

The MRS at the upper part is -2. The MRS in the lower part is -$\frac{1}{2}$. If market prices for the two goods are sufficiently different, we are in the same situation as with perfect substitutes preferences. If $\frac{p_1}{p_2} > 2$, then the optimal bundle is at the corner $(0, m/p_2)$. If $\frac{p_1}{p_2} < 2$ then the optimal bundle is at the corner $(m/p_1, 0)$. When p_1/p_2 is in between, the two arguments of the utility function must equate at any optimal choice. Otherwise the consumer would be spending too much on one good or the other.

Microeconomics (5SSPP217)

Seminar

Felipe Torres
felipe.torres@kcl.ac.uk

June 4, 2025

Contact & Office Hours

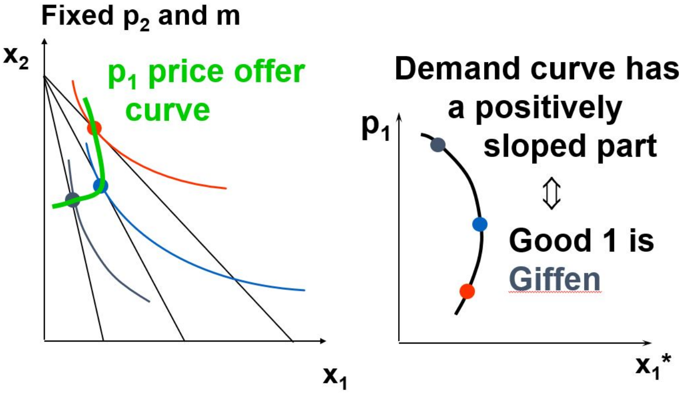
- My email: **felipe.torres@kcl.ac.uk**
- Office hours: Every Wednesday from 13:00 until 14:00. BH (SE)2.06

Intro

- For some values of its own-price, the quantity demanded of a good rises as its own-price increases then the good is called a **Giffen good**
- A good for which quantity demanded falls as income increases is called **inferior**

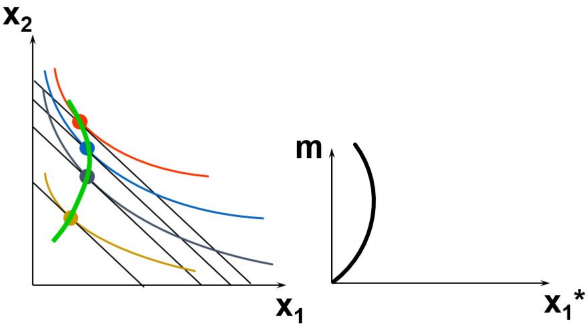
Intro

Giffen good



Intro

Inferior good



Intro

- We talked about the **elasticity**, which measures the *sensitivity* of one variable with respect to another

$$\varepsilon_{x,y} = \frac{\%\Delta x}{\%\Delta y} \tag{1}$$

Own-price Elasticity of Demand:

$$\varepsilon_{x_1,p_1} = \frac{\%\Delta x_1}{\%\Delta p_1} \tag{2}$$

Intro

Point Own-price Elasticity:

$$\varepsilon_{x_1,p_1} = \frac{p_1'}{Xi'} \cdot \frac{\partial X_i}{\partial p_i} \tag{3}$$

Point Own-price Elasticity: We are looking at the own-price elasticity **at the point (X_i', p_i')**

$$\varepsilon_{x_1,p_1} = \frac{p_1'}{Xi'} \cdot \frac{\partial X_i}{\partial p_i} \tag{4}$$

Intro

Point Own-price Elasticity:

- $-1 < \varepsilon_{x_1,p_1} < 0$ own-price inelastic
- $\varepsilon_{x_1,p_1} = -1$ own-price unit elastic
- $\varepsilon_{x_1,p_1} < -1$ own-price elastic

Intro

We also talked about ordinary demand functions and inverse demand functions:

- Ordinary demand function (Marshallian Demand Function): The quantity demanded as a function of price(s) $Q_x = AP_x + B$
- Inverse demand function: is the demand function viewing price as a function of quantity $P_x = \frac{Q_x}{A} - \frac{B}{A}$

Question 1, a

1. For the perfect substitutes utility function $U(x_1, x_2) = x_1 + x_2$ and the perfect complements utility function $U(x1, x2) = \min\{x_1, x_2\}$:

a) Write the expressions for their ordinary demands as a function of prices and income.

Question 1, a

Let's write the ordinary demand function for good x_1 for the perfect utility function $U(x_1, x_2) = x_1 + x_2$:

Remember that ordinary demand functions are asking, given p_1', what quantity is demanded of commodity 1? Thus, x_1 must be as a function of p_1, conditional on whether p_1 is equal, greater than or less than p_2.

$$x_1^* = \begin{cases} 0 \text{ if } p_1 > p_2 \\ 0 < x_1 < \frac{m}{p_1} \text{ if } p_1 = p_2 \\ \frac{m}{p_1} \text{ if } p_1 < p_2 \end{cases} \tag{5}$$

Question 1, a

1.a) First scenario:

- We have utility function with perfect substitutes
- Let's assume that p_1 is equal to p_1'
- Where $p_1' < p_2$
- We reached a corner solution $\frac{m}{p_1'}$

We reached a corner solution $MRS = 1 > p_1/p_2$, where we consume only $\frac{m}{p_1'}$.

$$x_1^*(p_1, p_2, m) = \frac{m}{p_1} \tag{6}$$

Question 1, a

1.a) Second scenario:

- Let's raise the price of $p_1 = p_1''$
- Where $p_1'' = p_2$
- We consume any quantity between 0 and $\frac{m}{p_1''}$ This means we will consume any amount in the budget line

$$x_1^*(p_1, p_2, m) = [0, \frac{m}{p_1}] \tag{7}$$

Question 1, a

1.a) Third scenario:

- Let's keep raising the price of p_1 to p_1'''
- Where $p_1''' > p_2$
- We reached again to a corner solution
- Where we consume $\frac{m}{p_2}$, and $x_1 = 0$

$$x_1^*(p_1, p_2, m) = 0 \tag{8}$$

Question 1, a

In summary:

$$x_1^*(p_1, p_2, m) = \begin{cases} 0 \text{ if } p_1 > p_2 \\ 0 < x_1 < \frac{m}{p_1} \text{ if } p_1 = p_2 \\ \frac{m}{p_1} \text{ if } p_1 < p_2 \end{cases} \tag{9}$$

Question 1, a

Now, let's find the ordinary demand for the complements utility function: $U(x_1, x_2) = \min\{x_1, x_2\}$. For $p_1 = p_1'$, where $p_1' < p_2$. Then, $x_1' > x_2$, then the consumer will consume x_2, and the optimal consumption will be $x_2 = x_1$

$$x_1 p_1 + x_2 p_2 = m \text{ where } x_1 = x_2 \tag{10}$$

$$x_1 p1 + x_2 p_2 = m \tag{11}$$

$$x_1(p_1 + p_2) = m \tag{12}$$

$$x_1^*(p_1, p_2, m) = \frac{m}{p_1 + p_2} \tag{13}$$

Question 1, a

Let's increase p_1 to p_1''. Where $p_1'' = p_2$. Then, the consumer will consume $x_1 = x_2$ and the optimal consumption bundle is again $x_1 = x_2$.

The ordinary demand function is the same, but we decrease our consumption of x_1, as p_2 and m is fixed, thus $x_*(p_1, p_2, m)$ decreases.

$$x_1^*(p_1, p_2, m) = \frac{m}{p_1 + p_2} \tag{14}$$

Question 1, a

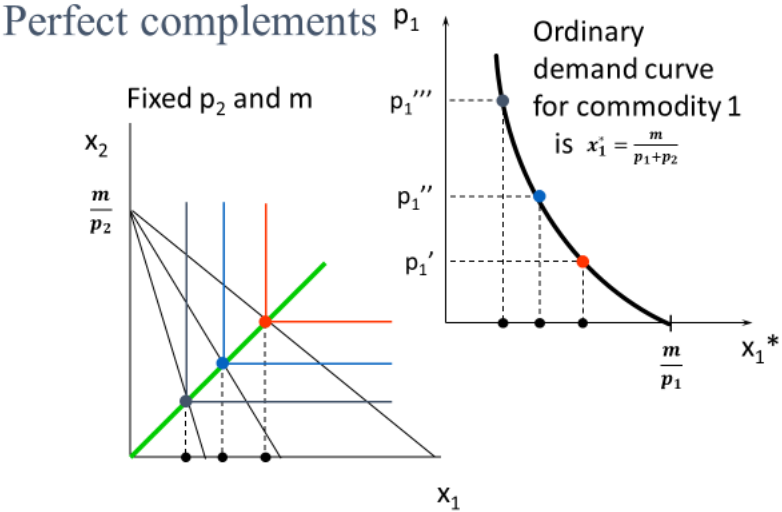
Let's increase again p_1 to p_1''', where $p_1''' > p_2$, then the consumer will consume less of x_1, but again will consume x_1''', which is the smallest amount between the two goods, but again the consumer will consume x_2, where $x_2 = x_1$.

The ordinary demand function is the same, but we decrease our consumption of x_1 even **further**, as p_2 and m is fixed, thus $x_*(p_1, p_2, m)$ decreases.

$$x_1^*(p_1, p_2, m) = \frac{m}{p_1 + p_2} \tag{15}$$

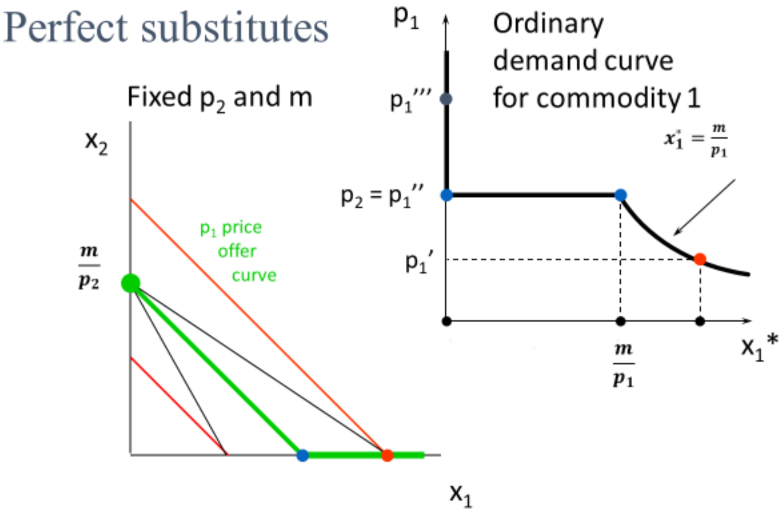
Question 1, b

Complementary goods:



Question 1, b

Substitute goods:

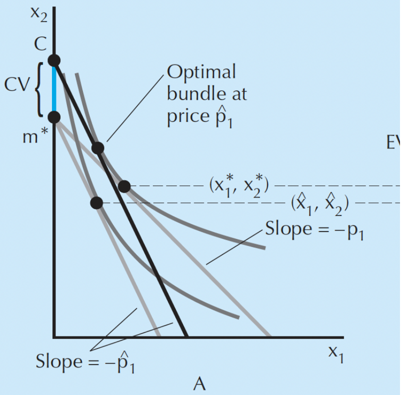


Question 2

The **compensating equivalent** is defined as the level of income a buyer/consumer needs to have in order to attain the same level of utility they had before a change in prices. Let's now go back to problem 5 in Problem Set 1 from last week. Suppose that the cost of cultural activities doubles. What is the compensating equivalent? In other words, what should be the new municipal budget B you would need if you want to ensure your citizens attain the same level of utility as before the change?[1]

[1] For more info on CE, see page 258 (Chapter 14), Hal Varian's book

Question 2



The compensating and the eq
A shows the compensating variati
the equivalent variation (EV).

Question 5, a

5. You are the major of a little town. Your budget is £38k and you must decide how much to allocate to fund sports facilities (good x) and cultural activities (good y). Each sport activity costs £2k and each cultural activity £1k. The preferences of your citizens over sports and cultural activities can be represented by the utility function $U(x, y) = 2\ln(x) + y$

a) Draw the budget constraint and a few indifference curves (make sure to derive the mathematical expression for the curves before drawing them).

Question 5, a

5.a) Draw the budget constraint:

We know that:

- Sport facilities x
- Cultural activities y
- Price sport facilities $p_x = 2$
- Price cultural activities $p_y = 1$, but now it doubles so $p_y = 2$
- Total budget £$38k$

Question 2

What is our goal:

- To calculate the compensating variation we ask how much money would be necessary at prices (2,2) to make the consumer as well of as she was consuming bundle (1,36).
- Stated mathematically, we are looking at $\upsilon(\hat{x}) + B = \upsilon(x^*)$, and solving for CV,
- where $\hat{x}$ is our consumption under the new set of prices, and x^* is the optimal bundle under the original prices

Question 2

Let's remember the utility function and budget constraint from question 5:
Utility function:

$$U(x, y) = 2\ln(x) - y \tag{16}$$

- Price of cultural activities $p_y = 1$
- Price of sport facilities $p_x = 2$

Budget constraint:

$$2x + y = 38 \tag{17}$$

The consumption bundle was $x^* = 1$ and $y^* = 36$.

Question 2

They tell us that the compensated equivalent is a condition where:

- Keep the same level of utility
- Under new prices p_x and p'_y
- So what is the new level of income B required to achieve that?

Question 2

Let's first calculate the level of utility achieved before prices changed:

$$U(x, y) = 2\ln(x) - y \Rightarrow U(1, 36) = 36 \tag{18}$$

- The price of cultural activities doubles $p_y = 1$ and $p'_y = 2$

Question 2

Now, we need to find our new bundle with the new price(s):

$$\begin{aligned} \max U(x,y) \quad & 2\ln(x) + y \\ \text{s.t.} \quad & 2x + 2y = B \end{aligned} \tag{19}$$

Let's get the ordinary demand

$$L(x; y; \lambda) = 2\ln(x) + y - \lambda(2x + 2y - B) \tag{20}$$

Question 2

We use the Lagrangian method:

$$L(x; y; \lambda) = 2\ln(x) + y - \lambda(2x + 2y - B) \tag{21}$$

FOCs:

$$\frac{\partial L}{\partial x} = \frac{2}{x} - 2\lambda = 0 \tag{22}$$

$$\frac{\partial L}{\partial y} = 1 - 2\lambda = 0 \tag{23}$$

$$\frac{\partial L}{\partial \lambda} = -2x - 2y + B = 0 \tag{24}$$

Question 2

Solving for this system of equations:

$$\lambda = \frac{1}{2} \tag{25}$$

$$\frac{2}{x} = 2\lambda \tag{26}$$

$$\frac{\cancel{2}}{x} = \cancel{2}\frac{1}{2} \Rightarrow \hat{x} = 2 \tag{27}$$

$$2 \cdot -2 - 2y + B = 0 \Rightarrow \hat{y} = \frac{B}{2} - 2 \tag{28}$$

Question 2

We can plug $\hat{x}$ and $\hat{y}$ into the utility function:

$$U(x, y) = 2\ln(x) - y \tag{29}$$

$$U(\hat{x}, \hat{y}) = 2\ln(x) - y \Rightarrow 2\ln(2) + \frac{B}{2} - 2 \tag{30}$$

In order to compute the **compensating equivalent** we need to equate our new utility function to the previous level of utility, which was 36. Basically what we are doing, is that we are setting our maximisation problem under the new prices as function of B.

$$36 = 2\ln(2) + \frac{B}{2} - 2 \tag{31}$$

$$\frac{B}{2} = 38 - 2\ln(2) \tag{32}$$

$$B = 72 - 4\ln(2) \tag{33}$$

Question 2 - another way

When you are working with quasilinear preferences, you can also obtain the compensating variation using the following equation:

$$v(\hat{x}_1) + m + C - \hat{p}_1\hat{x}_1 = v(x^*) + m - p_1^* x_1^* \tag{34}$$

Where $\hat{p}_1$ is the new price for good 1. p_1^* is the old price. x_1^* is the bundle consumed at the old price. $v(x^*)$ is the utility attained at price p_1^*. Finally m is the original budget, and C is the compensating variation. Rearranging this equation and get we C in one side:

$$C = v(x^*) - v(\hat{v}) + \hat{p}_1\hat{x}_1 - p_1^* x_1^* \tag{35}$$

Question 3

3. In the lectures, we saw the decomposition of the overall change in demand into the income and substitution effect for the case of a normal good. Using the same graphical method, draw the decomposition the overall change in the demand of an inferior good following a decrease in its price (with the inferior good in the horizontal axis).

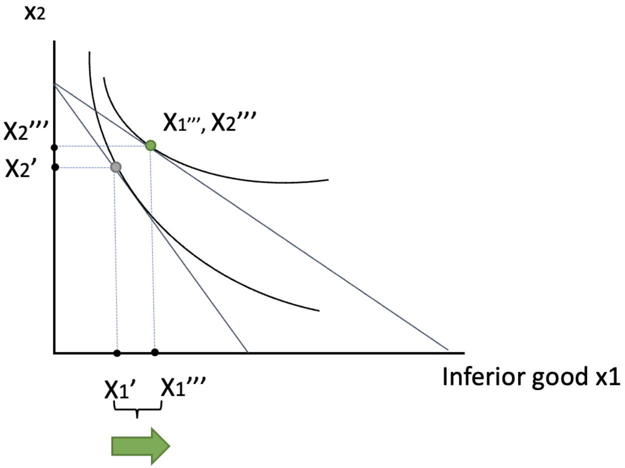
Question 3

Definition of an inferior good?

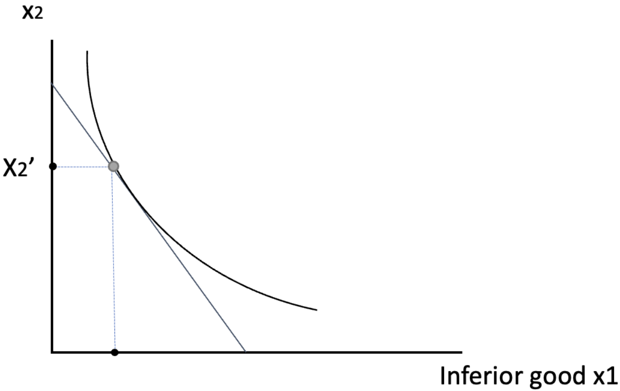
Question 3

Inferior good: As income increases, we consume less of that good.

Question 3

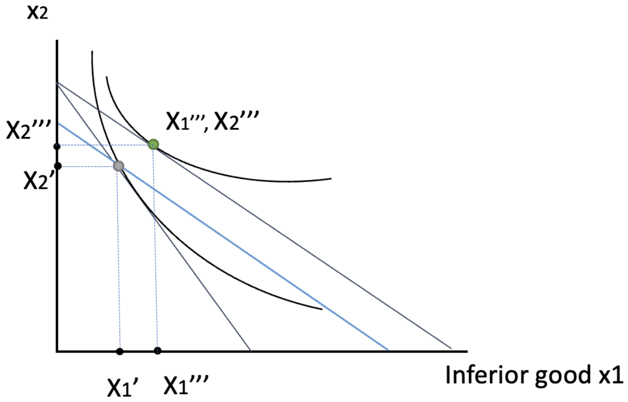


Question 3

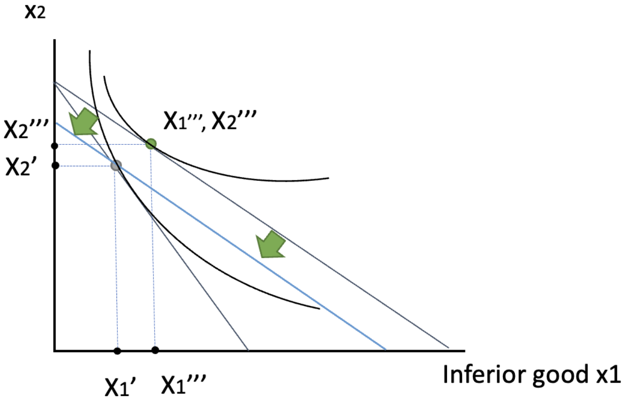


Question 3

To identify the pure income effect we need to find the substitution effect. And therefore, we need find a new budget line that keeps our original bundle affordable, but facing new relative prices.

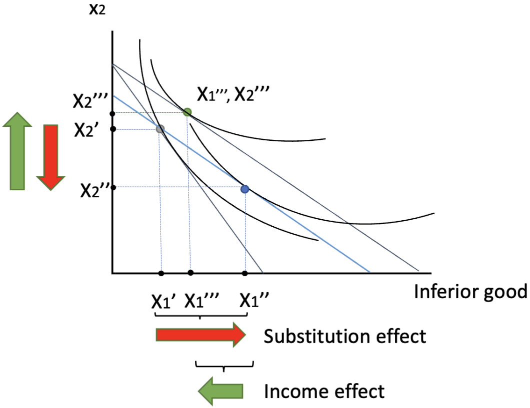


Question 3



Question 3

But for inferior goods, we know that whenever income increases, the demand for that good decreases. Here we can see real income increases, thus we should expect consumption of the inferior to decrease .

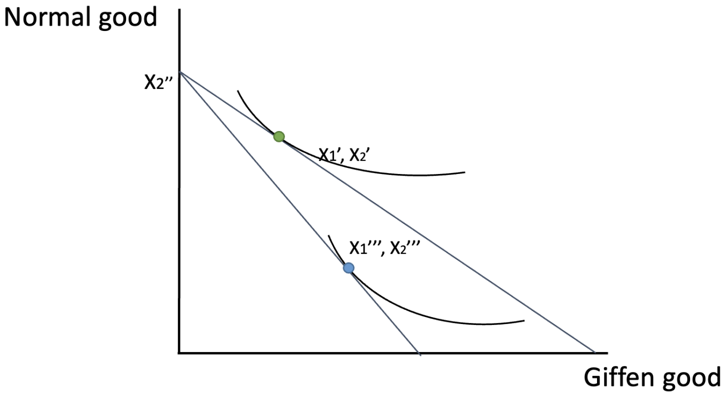


Question 4

4. Discuss in terms of the relative strength of the income and substitution effect and using the concepts of inferior/normal good why potatoes might have been a Giffen good during the Irish Great Famine.

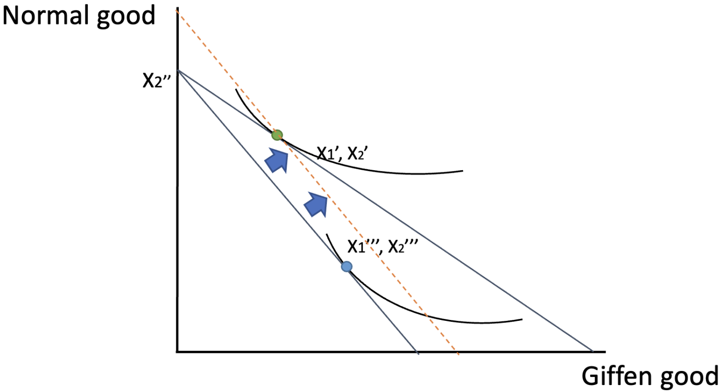
Question 4

4. Answer:



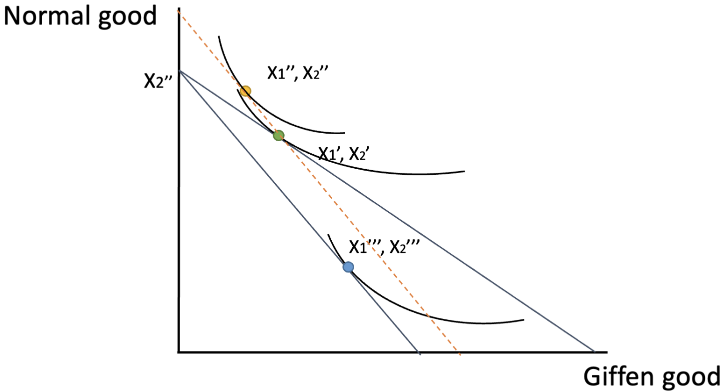
Question 4

4. Answer:



Question 4

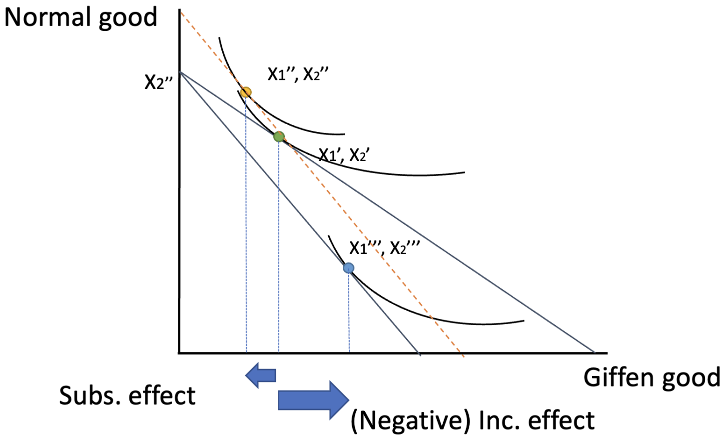
4. Answer:



The substitution effect push down the consumption of potatoes.

Question 4

4. Answer:



The income effect made people relatively poorer, but potatoes were probably an inferior good meaning that the negative income effect made people consume more of them.

Question 4

4. Answer:

If the income effect was stronger than the substitution effect, the net effect might have been an increase in demand despite the rise in prices. It helped that there were virtually no substitutes to potatoes on those days.

Question 5

The demand for widgets in Asia is $q^A(p) = 12–2p$. The demand for widgets in Europe is $q^E(p) = 9–p$.

a) Show that the price elasticity for the demand of widgets in Asia is -1.4 when the price is 3.5. If the price were to rise, would revenue rise or fall in that market?

b) Suppose widgets are not demanded anywhere else in the world. Compute the world demand of widgets and draw it.

Question 5, a

$$\varepsilon_p^A = \frac{p}{q^A}\frac{\partial q^A}{\partial p} \tag{36}$$

$$\varepsilon_p^A = \frac{p}{12-2p} \cdot -2 \tag{37}$$

$$\varepsilon_p^A = \frac{-2p}{12-2p} \Rightarrow = \frac{-p}{6-p} \tag{38}$$

For price $p = 3.5$

$$\varepsilon_p^A = \frac{-3.5}{6-3.5} \Rightarrow \frac{-3.5}{2.5} \tag{39}$$

$$\varepsilon_p^A = -1.4 \tag{40}$$

Question 5, a

$$\varepsilon_p^A = -1.4 \tag{41}$$

At that price, a 1% increase in the price is associated with a 1.4% decreased in demand for that good. At that point, the demand is elastic and the revenue of the seller would drop if the price were to rise.

Question 5, b

b) Suppose widgets are not demanded anywhere else in the world. Compute the world demand of widgets and draw it.

Question 5, b

b) How do we get the world demand?

Question 5, b

b) We basically add up both demands and we get the world demand for widgets, **but be careful!**

p	Qe	Qa	Qw
0	9	12	21
1	8	10	18
6	3	0	9
8	1	0	1
9	0	0	0

Table: World demand by price

Question 5, b

b) Alternatively, we can see that whenever the price of widgets is between 0 and 6, there is a demand both from Europe and Asia. In this case, the World's demand would be the addition of the two ordinary demand functions. (For prices $p < 6$)

$$q^A(p) = 12–2p \text{ and } q^E(p) = 9–p \tag{42}$$

$$Q(p) = q^A(p) + q^E(p) \tag{43}$$

$$Q(p) = 12–2p + 9–p \tag{44}$$

$$Q(p) = 21–3p \tag{45}$$

Question 5, b

b) When prices are between 6 and 9, there is only a demand for widgets from Europe. Thus, the world demand function is Europe's demand function. $(6 \leq p \leq 9)$.

$$Q(p) = q^E(p) \tag{46}$$

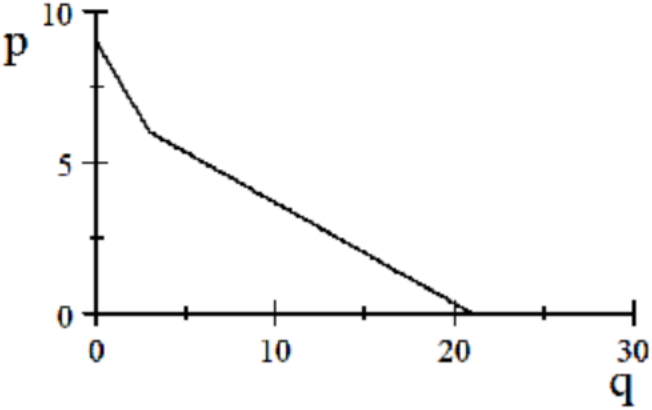
$$Q(p) = 9 - p \tag{47}$$

Question 5, b

b) We see that when p goes from 6 to 9, there is only a demand from Europe, thus there is a change of slope in the World demand. We can formally state this as follows:

$$Q(p) = \begin{cases} 21 - 3p \text{ if } p < 6 \\ 9 - p \text{ if } 6 \leq p \leq 9 \end{cases} \tag{48}$$

Question 5, b



Question 6

6. The demand curves for steak, eggs, and hot dogs are given in the table below. The current price of steak is $p_s = £5$. The price of eggs is $p_e = £2.50$, and the price of hot dogs is $p_h = £0.75$. Fill in the remaining columns of the table using this information. Indicate which goods are substitutes and which goods are complements.

Good	Demand Equation	Steak Price e_s_d	Egg Price e_e_d	Hotdog Price e_h_d
Steak	$D_S = 500 - 2P_S - \frac{1}{10}P_E + P_H$			
Egg	$D_E = 75 - 3P_E - P_S + \frac{1}{10}P_H$			
Hotdog	$D_H = 300 - \frac{1}{2}P_H + P_S + \frac{1}{10}P_E$			

Table: Problem 6

Question 6

6. Basically, they are asking to find the point own-price elasticity and the cross-price elasticity for these three goods:

$$\varepsilon_p = \frac{p}{q}\frac{\partial q}{\partial p} \tag{49}$$

Let's find the point own-price elasticity for steak:

$$\varepsilon_{D_s,p_s} = \frac{p_s}{D_s}\frac{\partial D_s}{\partial p_s} \tag{50}$$

$$\varepsilon_{D_s,p_s} = \frac{p_s}{500 - 2P_S - \frac{1}{10}P_E + P_H}\frac{\partial(500 - 2P_S - \frac{1}{10}P_E + P_H)}{\partial p_s} \tag{51}$$

$$\varepsilon_{D_s,p_s} = \frac{p_s}{500 - 2P_S - \frac{1}{10}P_E + P_H} \cdot -2 \tag{52}$$

Question 6

6. Basically, they are asking to find the own-price elasticity and the cross-price elasticity for these three goods:

$$\varepsilon_{D_s,p_s} = \frac{-2p_s}{500 - 2P_S - \frac{1}{10}P_E + P_H} \tag{53}$$

We can now plug in the different prices into our elasticity equation $p_s = 5$, $p_e = 2.5$ and $p_s = 0.75$:

$$\varepsilon_{D_s,p_s} = \frac{-2 \cdot 5}{500 - 2 \cdot 5 - \frac{1 \cdot 2.5}{10} + 0.75} \tag{54}$$

$$\varepsilon_{D_s,p_s} = -0.02 \tag{55}$$

Is this good elastic or inelastic?

Question 6

6. Before computing the cross-price elasticity between steak and eggs, let's understand the intuition behind the cross-price elasticity:

$$\varepsilon_{x,y} = \frac{p_y}{D_x}\frac{\partial Q_x}{\partial p_y} \tag{56}$$

$$\varepsilon_{x,y} = \frac{\text{Percentage change in Quantity of X}}{\text{Percentage change in Price of Y}} \tag{57}$$

$$\varepsilon_{x,y} = \frac{\frac{\Delta Q_x}{Q_x}}{\frac{\Delta P_y}{P_y}} \tag{58}$$

$$\varepsilon_{x,y} = \frac{\Delta Q_x}{Q_x} \cdot \frac{P_y}{\Delta P_y} \tag{59}$$

$$\varepsilon_{x,y} = \frac{\Delta Q_x}{\Delta P_y} \cdot \frac{P_y}{Q_x} \tag{60}$$

Question 6

6. Before computing the cross-price elasticity between steak and eggs, let's understand the intuition behind the cross-price elasticity:

$$\varepsilon_{x,y} = \frac{\Delta Q_x}{\Delta P_y} \cdot \frac{P_y}{Q_x} \tag{62}$$

$$\varepsilon_{x,y} = \frac{\Delta Q_x}{\Delta P_y} \cdot \frac{P_y}{Q_x} \Rightarrow \frac{p_y}{D_x} \frac{\partial Q_x}{\partial p_y} \tag{63}$$

Question 6

6. Now we compute the cross-price elasticity of steaks with respect to eggs and hot dogs. Let's calculate first the cross-price elasticity between steak and eggs:

$$\varepsilon_{x,y} = \frac{p_y}{D_x}\frac{\partial Q_x}{\partial p_y} \tag{64}$$

In our case, x = steak, y = eggs:

$$\varepsilon_{s,p_e} = \frac{p_e}{D_s}\frac{\partial D_s}{\partial p_e} \tag{65}$$

Question 6

6. Now we compute the cross-price elasticity of demand between steaks and eggs.

$$\varepsilon_{s,p_e} = \frac{p_e}{500 - 2P_s - \frac{1}{10}P_e + P_h} \cdot \frac{\partial(500 - 2P_s - \frac{1}{10}P_e + P_h)}{\partial p_e} \tag{66}$$

$$\varepsilon_{s,p_e} = \frac{p_e}{500 - 2P_s - \frac{1}{10}P_e + P_h} \cdot \frac{-1}{10} \tag{67}$$

We can now plug in the different prices into our elasticity equation $p_s = 5$, $p_e = 2.5$ and $p_s = 0.75$:

$$\varepsilon_{s,p_e} = \frac{2.5}{500 - 2 \cdot 5 - \frac{1 \cdot 2.5}{10} + 0.75} \cdot \frac{-1}{10} \tag{68}$$

$$\varepsilon_{s,p_e} = \frac{2.5}{490.5} \cdot \frac{-1}{10} \Rightarrow \varepsilon_{s,p_e} = -0.00051 \tag{69}$$

Question 6

6. Finally, let's compute the cross-price elasticity between steaks and hot dogs:

$$\varepsilon_{s,p_h} = \frac{p_h}{D_s}\frac{\partial D_s}{\partial p_h} \tag{70}$$

$$\varepsilon_{s,p_h} = \frac{p_h}{500 - 2P_s - \frac{1}{10}P_e + P_h} \cdot \frac{\partial(500 - 2P_s - \frac{1}{10}P_e + P_h)}{\partial p_h} \tag{71}$$

$$\varepsilon_{s,p_h} = \frac{p_h}{500 - 2P_s - \frac{1}{10}P_e + P_h} \cdot 1 \tag{72}$$

We can now plug in the different prices into our elasticity equation $p_s = 5$, $p_e = 2.5$ and $p_s = 0.75$:

$$\varepsilon_{s,p_h} = 0.0015 \tag{73}$$

Question 6

6. Finally, let's compute the cross-price elasticity between steaks and hot dogs:

$$\varepsilon_{D_s,p_s} = -0.02 \Rightarrow \text{Inelastic} \tag{74}$$

$$\varepsilon_{D_s,p_e} = -0.00051 \Rightarrow \text{Steak and eggs are complements} \tag{75}$$

$$\varepsilon_{D_s,p_h} = 0.00153 \Rightarrow \text{Steak and hot dogs are substitutes} \tag{76}$$

Question 6

Good	Demand Equation	Steak Price e_s_d	Egg Price e_e_d	Hotdog Price e_h_d
Steak	$D_S = 500 - 2P_S - \frac{1}{10}P_E + P_H$	-0.020	-0.00051	0.00153
Egg	$D_E = 75 - 3P_E - P_S + \frac{1}{10}P_H$	-0.079	-0.120	0.0012
Hotdog	$D_H = 300 - \frac{1}{2}P_H + P_S + \frac{1}{10}P_E$	0.016	0.00082	−0.0012

Table: Problem 6 - Solutions

Question 7, a)

Consider an economy with only two consumers whose utility functions are those in question 1 (that is , one consumer has perfect complement preferences and the other has perfect substitute preferences). Suppose that both consumers have the same income m $=$ 10 and assume the price of good 2 is $p_2 = 1$
a) Find the mathematical expression for the market demand for good 1. Draw it.

Question 7, a)

The market demand is the horizontal (quantity) sum of individuals demands. Note that for $p_1 > 1$the perfect substitute consumer will not demand good 1 so above that price the market demand is just the demand of the perfect complements consumer:

$$x^*_{1,PC} = \frac{10}{p_1 + p_2} \text{ if } p > 1 \tag{77}$$

Question 7, a)

If $p_1 = 1$
for the consumer with perfect substitute:

$$x^*_{1,PS} = 0 < x_1 < \frac{m}{p_1} if p_1 = p_2, \text{ thus} \tag{78}$$

$$x^*_{1,PS} = [0, 15] \tag{79}$$

for the consumer with perfect complements:

$$x^*_{1,PC} = \frac{m}{p_1 + p_2} \text{ thus} = \frac{10}{1+1} \longrightarrow 5 \tag{80}$$

Thus, the total demand goes from 5 to 15.
$X^*_1 = x^*_{1,PC} + x^*_{1,PS} = [5, 15]$

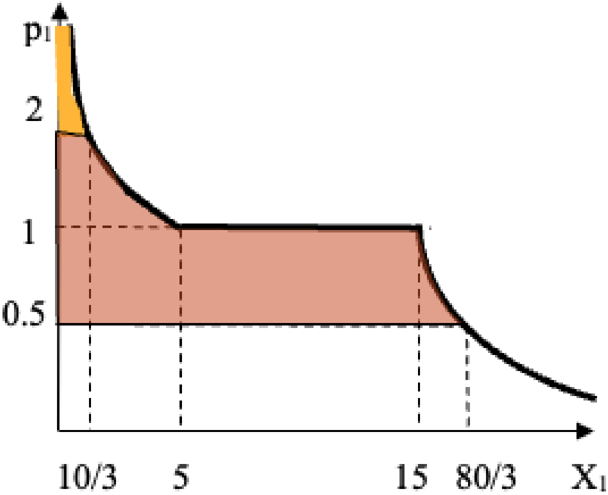
Question 7, a)

If $p_1 < 1$

$$X_1^* = \frac{10}{p_1 + 1} + \frac{10}{p_1} = \frac{10(1 + 2p_1)}{(p1(1 + p_1))} \tag{81}$$

Question 7, a)

Graphically (not scaled).



Question 7, b)

b) Using the market demand, write the integral measuring the total consumer surplus when:

1. The price of good is $p_1 = 2$. For this one, integrate across quantities.
2. The price of good is $p_1 = 0.5$. For this one, integrate across prices.

You do not to solve these integrals, just write them up.

Question 7, b)

If $p_1 = 2$, then, the demand for PS consumer $p_1 > p_2$ $(p_2 = 1)$

$$x^*_{1,PS} = 0 \tag{82}$$

$$x^*_{1,PC} = \frac{10}{p_1 + p_2} \text{ if } p > 1 \tag{83}$$

So the total consumer surplus is the same as the surplus for the perfect complements consumer. Since we asked to integrate across quantities, we need to find first the inverse demand for that consumer.

$$x^*_{1,PC} = \frac{10}{p_1 + p_2} \longrightarrow x^*_{1,PC} = \frac{10}{p_1 + 1} \longrightarrow p_1 - 1 = \frac{10}{x_1} \tag{84}$$

Question 7, b)

$$p_1 + 1 = \frac{10}{x_1} \tag{85}$$

$$p_1 = \frac{10}{x_1} - 1 \tag{86}$$

$$CS = \int_0^{\frac{10}{3}} (\frac{10}{x_1} - 1) - 2\, dx_1 \tag{87}$$

We get the $\frac{10}{3}$ from the ordinary demand function by plugging $p_1 = 2$

$$x^*_{1,PC} = \frac{10}{p_1 + p_2} \longrightarrow \frac{10}{2+1} \longrightarrow \frac{10}{3} \tag{88}$$

Question 7, b)

b) Using the market demand, write the integral measuring the total consumer surplus when:

1. The price of good is $p_1 = 2$. For this one, integrate across quantities.
2. The price of good is $p_1 = 0.5$. For this one, integrate across prices.

You do not to solve these integrals, just write them up.

Question 7, b)

If $p_1 = 0.5$, then, the demand comes from both consumers $p_1 < p_2$ $(p_2 = 1)$.

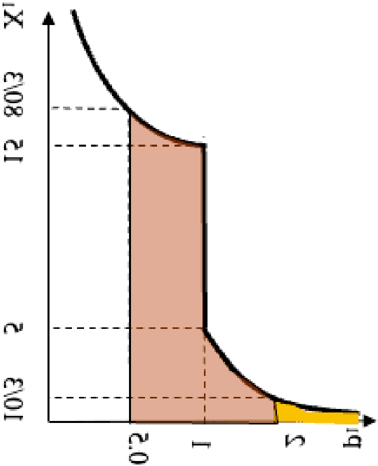
$$X_1^* = \frac{10(1+2p_1)}{(p1(1+p_1))} \text{ between} p_1 = 0.5 \text{ and} p_1 = 1 \tag{89}$$

The second part of the CS comes from the PS consumer that goes from 1 to ∞

In this case, we can use use the ordinary demand function:

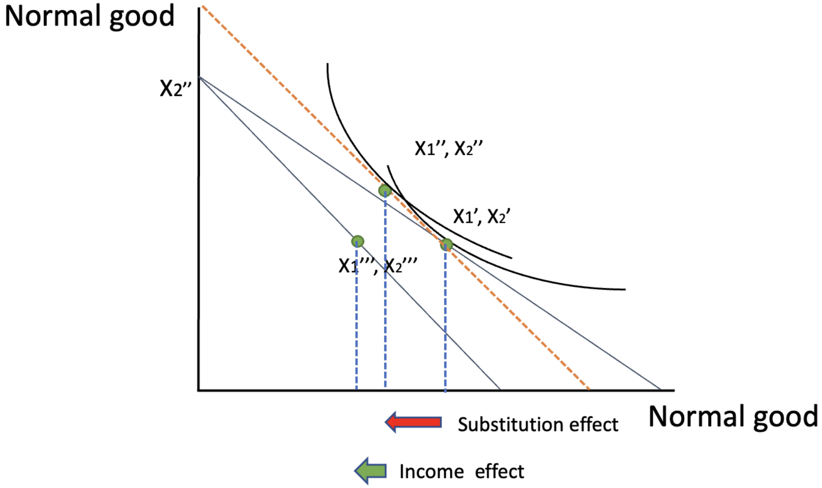
$$CS = \int_{0.5}^{1} \frac{10(1+2p_1)}{(p1(1+p_1))} \, dp_1 \ + \int_{1}^{\infty} \frac{10}{p_1+1} \, dp_1 \tag{90}$$

Question 7, b)



Substitution and Income Effects for a normal good

What happens the price of a normal good increases:



Substitution and Income Effects for a normal good

We see both effects are in the same direction. Substitution effects reduces our consumption of the normal good as it's now more expensive. And with the current prices, our purchasing power is smaller, thus, we consume less of the normal good.

Microeconomics (5SSPP217)

Seminar

Felipe Torres
felipe.torres@kcl.ac.uk

June 4, 2025

Contact & Office Hours

- My email: **felipe.torres@kcl.ac.uk**
- Office hours: Every Wednesday from 13:00 until 14:00. BH.SE 2.07.
- But next week: Online or in-person: Friday - 12:00 - 13:00. BH.NE. 9.01

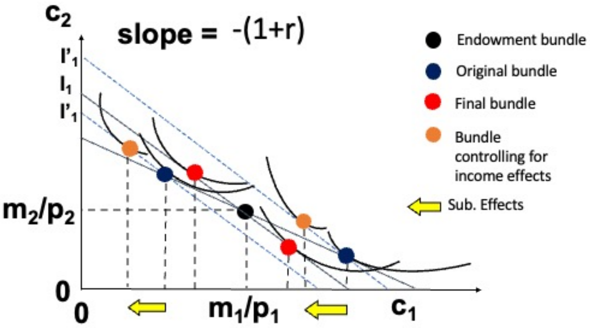
Question 1, a

1. Consider a consumer who lives two periods and for who current and future consumption are normal goods. The interest rate is r.

a) Suppose that the interest rate increases to r'. How would the consumer's budget constraint change? How current and future consumption will change due to the substitution effect?

Question 1, b

Substitution effect: Always negative



Question 1, b

The budget constraint will pivot around the endowment bundle, becoming steeper. The substitution effect will be to increase future consumption since its relative price decreased. Similarly, the substitution effect will be to decrease current consumption since its relative price has increased.

Question 1, b

b) If the consumer was saving before the change, how the income effect changes current and future consumption after the change in interest rate? You can draw the scenario to help you answer the question, but you must also use economic reasoning.

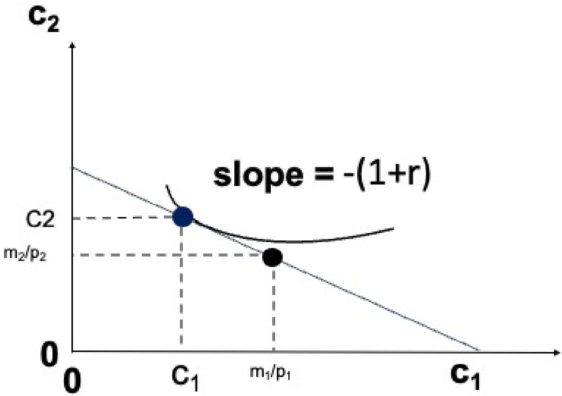
Question 1, b

Substitution and income effect in the case of a lender $c_1 < m_1$:

- The effect of the lender is ambiguous
- The substitution effect is **always** negative for a normal good
- An increase in the interest rate may give him so much extra income that he will want to consume even more in the first period.
- Alternatively, he could save even more and consume less in period 1. (Varian, p.189)

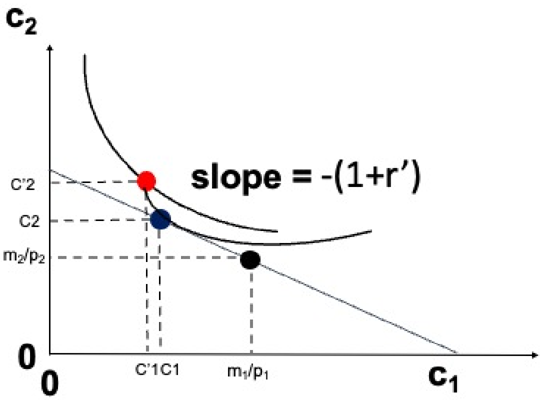
Question 1, b

Lender:



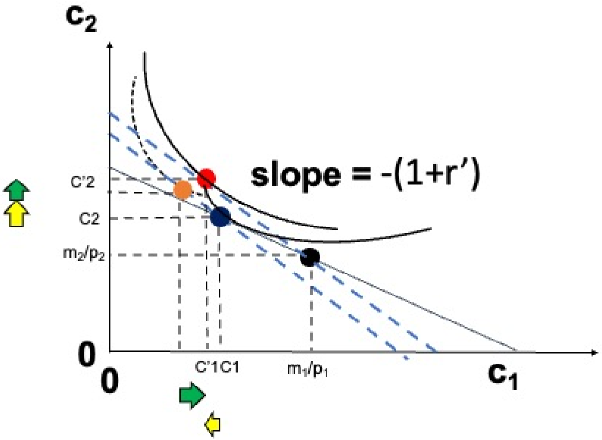
Question 1, b

Lenders - Scenario 1



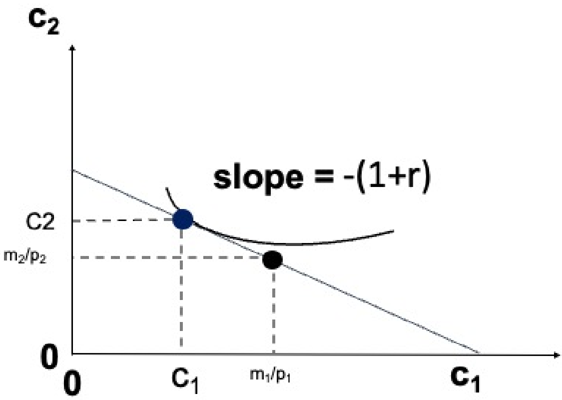
Question 1, b

Lenders - Scenario 1:



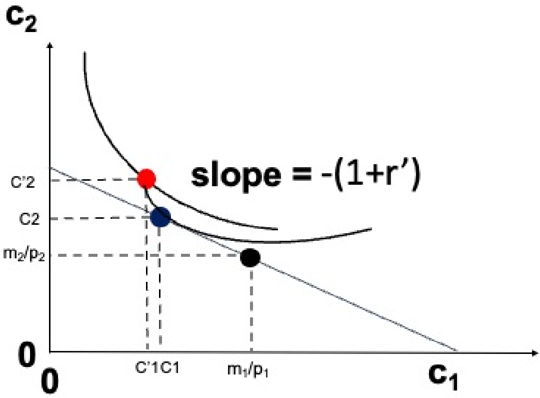
Question 1, b

Lenders - Scenario 2:



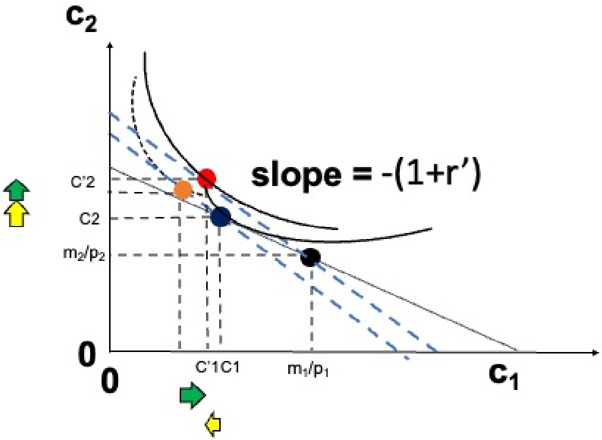
Question 1, b

Lenders - Scenario 2:



Question 1, b

Lenders - Scenario 2:

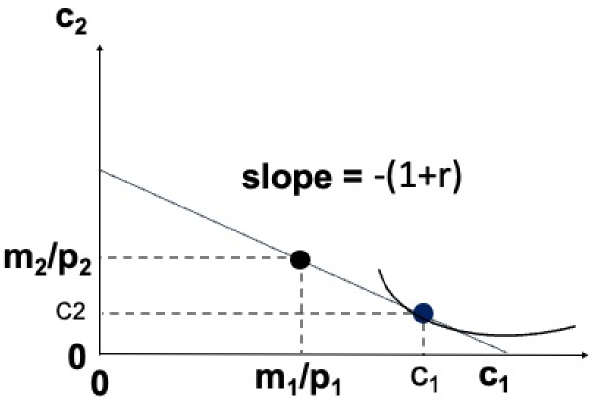


Question 1, c

c) Repeat b) assuming now the consumer was borrowing before the change

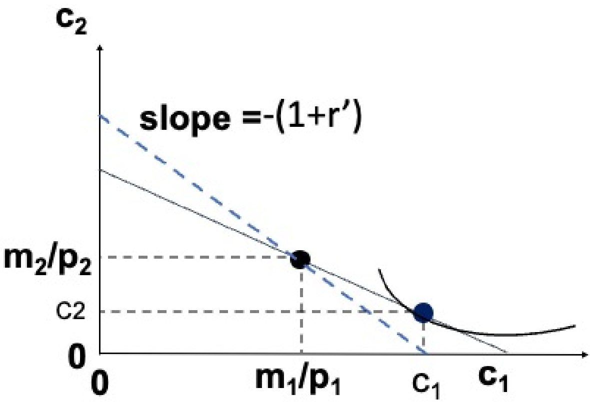
Question 1, c

Borrower - Scenario 1



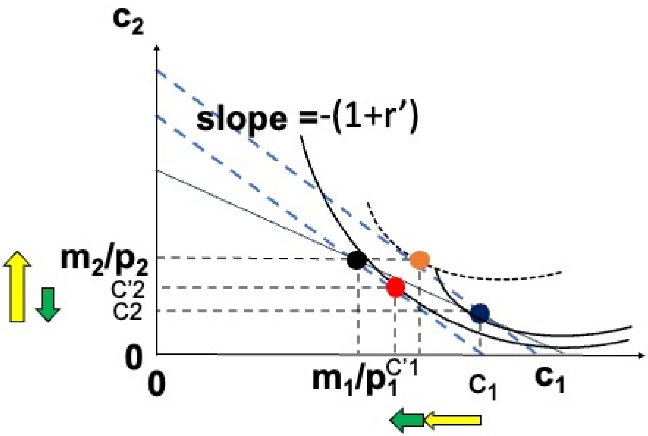
Question 1, c

Borrower - Scenario 1



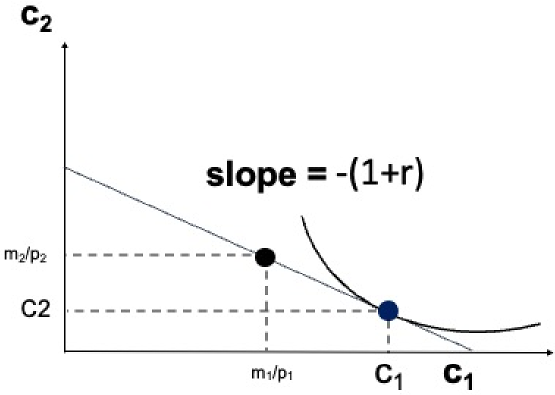
Question 1, c

Borrower - Scenario 1



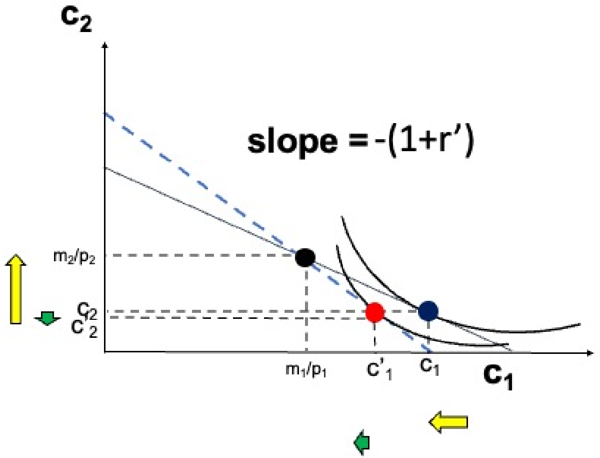
Question 1, c

Borrower - Scenario 2



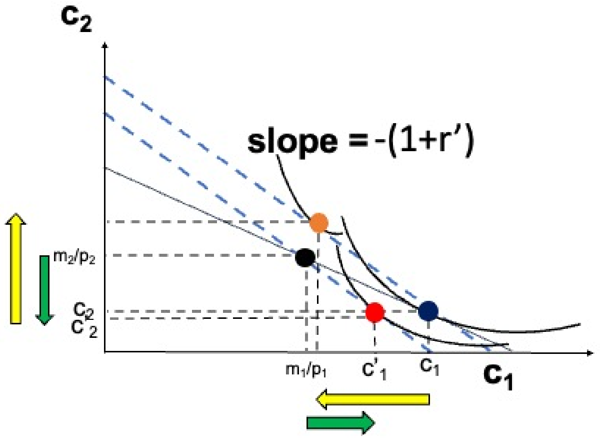
Question 1, c

Borrower - Scenario 2



Question 1, c

Borrower - Scenario 2



Question 1, c

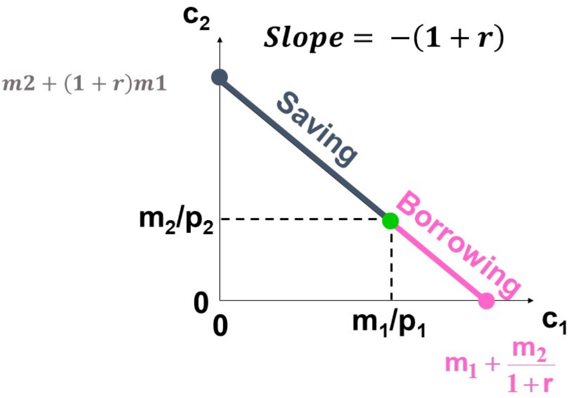
Substitution and income effect in the case of a borrower $c_1 > m_1$:

- Substitution effect always works opposite the direction of the price
- When the interest rises, there is always a substitution effect towards consuming less today
- It means that you will have to pay more interest tomorrow, which induces to borrow less (Varian, p.189)

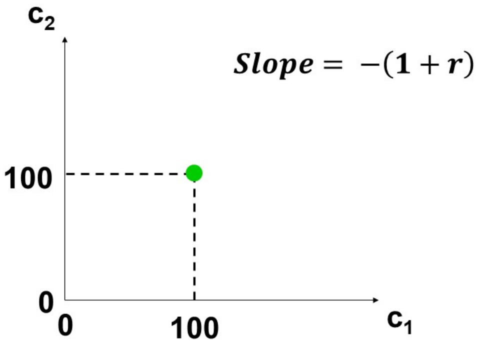
Question 2, a

Frank the farmer lives two periods. In the first period Frank earns £100 and the same in period 2. Savings are rewarded with a 20% interest rate whereas loans face a 10% interest rate.
a) Draw Frank's budget constraint specifying very clearly the intercept and the slope(s).

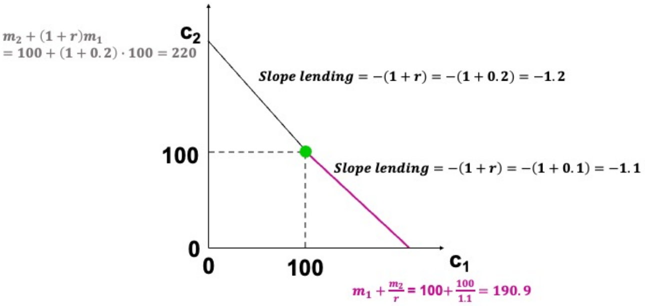
Question 2, a


c_2
$Slope = -(1+r)$
$m2 + (1+r)m1$
Saving
Borrowing
m_2/p_2
0
0
m_1/p_1
c_1
$m_1 + \frac{m_2}{1+r}$

Question 2, a



Question 2, a


$m_2 + (1+r)m_1$
$= 100 + (1+0.2)\cdot 100 = 220$
c_2
Slope lending $= -(1+r) = -(1+0.2) = -1.2$
100
Slope lending $= -(1+r) = -(1+0.1) = -1.1$
0
0
100
c_1
$m_1 + \frac{m_2}{r} = 100 + \frac{100}{1.1} = 190.9$

Question 2, a

b) Suppose now that Frank moonlights as an informal banker. The money he borrows against his farming income at a 10% interest rate he can later lend at a 20% interest rate to other farmers who do not have access to formal loans as he does. Draw Frank's new budget constraint specifying the intercept and the slope.

Question 2, b

b) Let's see what happens with the horizontal axis

- Does it change the intercept?

Question 2, b

b) Let's see what happens with the horizontal axis

- He can still consume all of his 1st period income, plus whatever then borrows from period 2 at a rate %10
- If he consumes all his income + whatever borrows then:
- Thus our intercept for c_1 keeps being the same 190.9

Question 2, b

b) Let's see what happens with the vertical axis:

- If he borrows his entire second period income at a 10% $(\frac{100}{1.1})$
- And borrows it to other farmers at a 20% $1.2 \cdot \frac{100}{1.1}$
- He will be able to obtain $(1.2) \cdot \frac{100}{1.1} = 190.9$
- Thus, the vertical intercept is:

$$1.2 \cdot 100 + 1.2\frac{100}{1.1} = 229.1 \tag{1}$$

Question 2, b

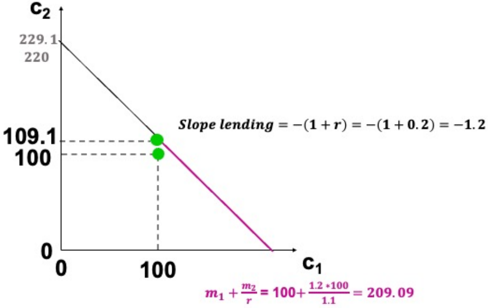
b) This allows to see better that the budget line is now a straight line with slope -1.2.

$$Slope = \frac{y_2 - y_1}{x_2 - x_1} \tag{2}$$

$$Slope = \frac{229 - 0}{0 - 190.9} \tag{3}$$

$$Slope = -1.2 \tag{4}$$

Question 2, b



c2
229.1
220
109.1
100
0
0
100
c1
Slope lending = −(1 + r) = −(1 + 0.2) = −1.2
m1 + m2/r = 100+ 1.2 *100 / 1.1 = 209.09

Question 3

In an isolated mountain village, the only crop is corn. Villagers plan for two time periods. In the first time period each villager will harvest 100 bushels. In the second time period, no corn will be harvested. There is no trade with the rest of the world and no stocks of corn remain from before the first period. Corn can be stored from one time period to the next, but rats eat 25% of what is stored. The villagers all have Cobb-Douglas utility functions $U(c_1, c_2) = c_1 c_2$ and can allocate their own corn between consumption and storage as they wish.

Question 3, a

a) Draw the budget constraint and the wealth endowment of these villagers. Using the Lagrange method, find the solution to their inter-temporal consumption problem.

Question 3, a

What would be the farmers endowment?

$$(100, 0)$$

Question 3, a

If villagers would store all their corn, they will be able to consume 75 bushels in period 2 as the rats would eat the rest:

$$100 - 0.25 \cdot 100 = 75 \tag{5}$$

$$(0, 75)$$

Question 3, a

If villagers would store all their corn, they will be able to consume 75 bushels in period 2 as the rats would eat the rest:

$$100 - .25 \cdot 100 = 75 \tag{6}$$

Thus our intercept in the vertical axis would be 75.

Question 3, a

What about the slope of our budget constraint? We can write our equation as the **present value format**:

$$c_1 + \frac{c_2}{1+r} = m_1 + \frac{m_2}{1+r} \tag{7}$$

$$c_1 + \frac{c_2}{1-0.25} = m_1 + \frac{0}{1-0.25} \tag{8}$$

$$c_1 + \frac{c_2}{0.75} = 100 \tag{9}$$

$$\frac{c_2}{0.75} = 100 - c_1 \tag{10}$$

$$c_2 = 75 - 0.75 \cdot c_1 \tag{11}$$

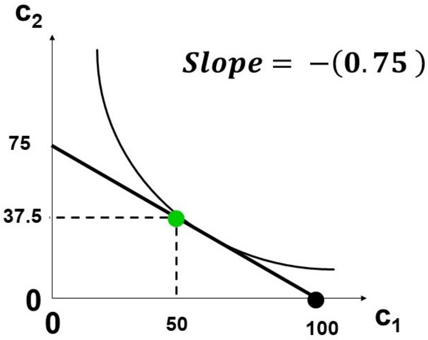
Question 3, a

If $c_1 = 0$, you consume $c_2 = 75$

$$c_2 = 75 - 0.75 \cdot c_1 \tag{12}$$

If $c_1 = 100$, you consume $c_2 = 0$, thus our slope is -0.75

Question 3, a



Question 3, a

$$\max U(c_1, c_2) = c_1 \cdot c_2 \quad \text{s.t.} \quad c_1 + \frac{c_2}{0.75} = 100 \tag{13}$$

We use the Lagrangian method:

$$L(c_1; c_2; \lambda) = c_1 \cdot c_2 - \lambda(c_1 + \frac{c_2}{0.75} - 100) \tag{14}$$

FOCs with respect to c_1, c_2, and λ:

$$\frac{\partial L}{\partial c_1} = c_2 - \lambda \cdot 0.75 = 0 \tag{15}$$

$$\frac{\partial L}{\partial c_2} = c_1 - \lambda = 0 \tag{16}$$

$$\frac{\partial L}{\partial \lambda} = -c_1 - \frac{c_2}{0.75} + 100 = 0 \tag{17}$$

Question 3, a

Second FOC with respect to c_2 :

$$\frac{\partial L}{\partial c_2} = c_1 - \lambda = 0 \Rightarrow c_1 = \lambda \tag{18}$$

We plug c_2 into the FOC with respect to c_1

$$\frac{\partial L}{\partial c_1} = c_2 - c_1 \cdot 0.75 = 0 \Rightarrow c_2 = c_1 \cdot 0.75 \tag{19}$$

$$\frac{\partial L}{\partial \lambda} = -c_1 - \frac{c_2}{0.75} + 100 = 0 \tag{20}$$

$$-c_1 - \frac{0.75c_1}{0.75} + 100 = 0 \tag{21}$$

$$-c_1 - \frac{\cancel{0.75}c_1}{\cancel{0.75}} + 100 = 0 \Rightarrow 2c_1 = 100 \tag{22}$$

$$c_1* = 50 \text{ and } c_2* = 37.5 \tag{23}$$

Question 3, a

We plug in the optimal consumption level into the utility function:

$$c_1^* = 50 \text{ and } c_2* = 37.5 \tag{24}$$

$$U(c_1, c_2) = c_1 \cdot c_2 \Rightarrow 37.5 \cdot 50 \tag{25}$$

$$U(c_1, c_2) = 1875 \tag{26}$$

Question 3, b

b) Suppose that the introduction of cats to the village reduces the rats' predations to 10% of what is stored. Draw the new budget constraint and find analytically the solution to the new inter temporal consumption problem of the villagers. Are they better off or worse off after the introduction of cats? (try to answer this last question without making any computation)

Question 3, b

Now r rather than being equal to 0.75, now r is equal to 0.90.

Question 3, b

$$\max U(c_1, c_2) = c_1 \cdot c_2 \quad \text{s.t.} \quad c_1 + \frac{c_2}{0.90} = 100 \tag{27}$$

We use the Lagrangian method:

$$L(c_1; c_2; \lambda) = c_1 \cdot c_2 - \lambda(c_1 + \frac{c_2}{0.90} - 100) \tag{28}$$

FOCs with respect to c_1, c_2, and λ:

$$\frac{\partial L}{\partial c_1} = c_2 - \lambda \cdot 0.90 = 0 \tag{29}$$

$$\frac{\partial L}{\partial c_2} = c_1 - \lambda = 0 \tag{30}$$

$$\frac{\partial L}{\partial \lambda} = -c_1 - \frac{c_2}{0.90} + 100 \tag{31}$$

Question 3, b

$$c_2 = \lambda \cdot 0.90 \tag{32}$$

$$c_1 = \lambda \tag{33}$$

$$c_2 = 0.90c_1 \tag{34}$$

We plug c_2 as a function of c_1 into the third FOC.

$$-c_1 - \frac{c_2}{0.90} + 100 \tag{35}$$

$$-c_1 - \frac{c_1.0.90}{0.90} + 100 \tag{36}$$

$$c_1 = 50. \tag{37}$$

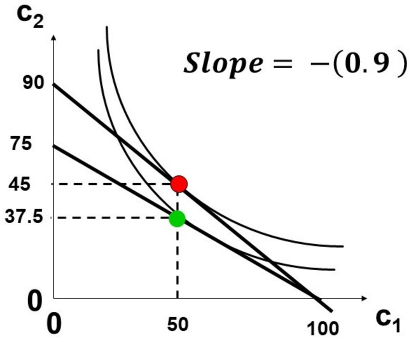
Question 3, b

$$c_1^* = 50 \text{ and } c_2^* = 45 \tag{38}$$

$$U(c_1^*, c_2^*) = c_1 \cdot c_2 \Rightarrow 50 \cdot 45 \tag{39}$$

$$2250 > 1875 \tag{40}$$

Question 3, b



Question 3, b

b) **Answer:**
Now, if villagers want to store all their corn, they will be able to consume 90 bushels in period 2. The new slope is thus 0.90 and the equation for the budget constraint is $c_1 + c_2/0.9 = 100$. The optimal consumption combination is $c_1 = 50$ and $c_2* = 45$ The villagers' utility level is 2250. Note that the villagers are net lenders (they transfer their consumption to the future) and that the cats are a form of increase in the interest rate (villagers can transfer more resources from the present to the future), hence it was to be expected that villagers would benefit from having the cats.

Question 4, a)

Consider three lotteries. In Lottery 1 you can earn £600 with 3% chance and 0 otherwise. With Lottery 2 you can earn £100 with 18% probability and 0 otherwise. With Lottery 3 you can earn £36 with 50% chance and 0 otherwise.

a) Compute the EMV of each lottery. Which one would a risk neutral person choose?

Question 4, a)

They asked us to compute the Expected Monetary Value - Lottery 1:

$$EMV = p_1 \cdot MV_1 + p_2 \cdot MV_2 \quad (41)$$

$$EMV = 0.03 \cdot 600 + (1 - 0.03) \cdot 0 \quad (42)$$

$$\pounds EMV = 0.03 \cdot \pounds 600 + 0.97 \cdot \pounds 0 \quad (43)$$

$$\pounds EMV = \pounds 18 \quad (44)$$

Question 4, a)

Lottery 2:

$$\pounds EMV = 0.18 \cdot \pounds 100 + 0.82 \cdot \pounds 0 \tag{45}$$

$$\pounds EMV = \pounds 18 \tag{46}$$

Lottery 3:

$$\pounds EMV = 0.50 \cdot \pounds 36 + 0.50 \cdot \pounds 0 \tag{47}$$

$$\pounds EMV = \pounds 18 \tag{48}$$

Question 4, a

a) Which one would a risk neutral person choose?

Question 4, a

a) Which one would a risk neutral person choose?

Answer: A risk neutral person would be indifferent between lotteries as they achieve the same monetary reward

Question 4, b

b) Suppose the person is risk averse. Draw the lotteries and the expected utility that person would derive from each of them in the space wealth-utility. Which one would the risk averse person take? Why?

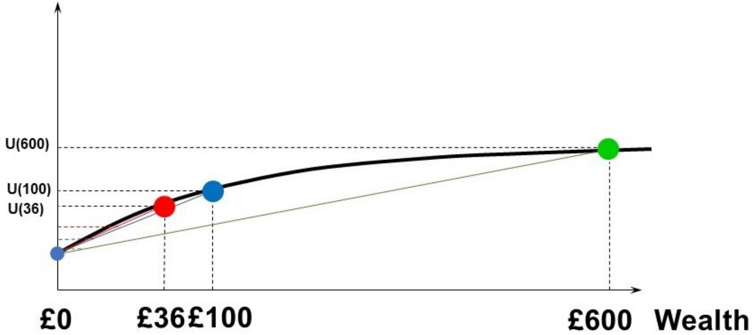
Question 4, b

First, let's compute the Expected Utility for each lottery:

- Lottery 1: $EU_1 = 0.03 \cdot U(600) + 0.97 \cdot U(0)$
- Lottery 2: $EU_2 = 0.18 \cdot U(100) + 0.82 \cdot U(0)$
- Lottery 3: $EU_3 = 0.50 \cdot U(36) + 0.50 \cdot U(0)$

Question 4, b

Let's draw the lotteries and expected utility for a risk-adverse person:

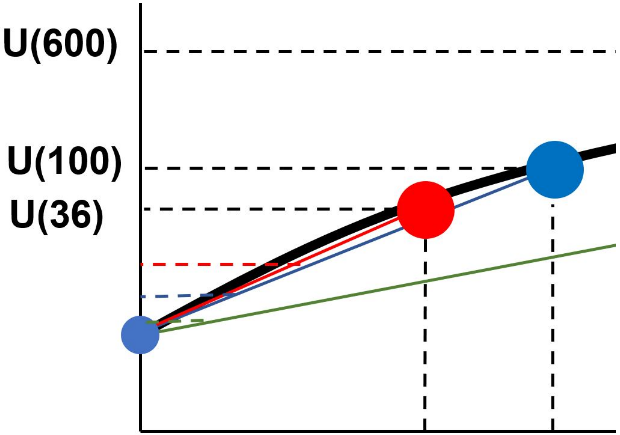


Question 4, b

Which one would the risk averse person take? Why?
Answer: Note that the EMV = £18 for all three lotteries. Risk aversion implies a concave utility function. It is graphically easy to see that Lottery 3 would provide the risk averse person the highest expected utility.

Alternatively to the graphical solution, since the EMV's coincide, the variance of each distribution can be used to rank the lotteries according to risk. The one with the lowest spread is Lottery 3.

Question 4, b



Question 5, a

Sandra lives in West Scotland and enjoys walking to and from work. Her utility is sharply diminished if she must walk while it is raining. Sandra's utility function is $U = 1{,}000$ if she walks and there is no rain, $U = 250$ if she drives to work instead and $U = 1$ if she walks to work and it rains. Sandra believes that the probability of rain today is $3/10$.

a) Given her beliefs, what is her expected utility from walking to work? What is her expected utility from driving to work according to her beliefs? If Sandra maximises her expected utility according to her beliefs, will she drive or walk to work?

Question 5, a

a) Given her beliefs, what is her expected utility from walking to work?

$$EU(Walking) = \frac{3}{10} \cdot 1 + (1 - \frac{1}{3}) \cdot 1000 \tag{49}$$

$$EU(Walking) = \frac{3}{10} \cdot 1 + (\frac{7}{10}) \cdot 1000 \tag{50}$$

$$EU = 700.3 \tag{51}$$

Question 5, a

a) What is her expected utility from driving to work according to her beliefs?

$$EU(Driving) = 1 \cdot 250 + 0 \cdot 1 \tag{52}$$

$$EU(Driving) = 250 \tag{53}$$

Question 5, a

a) If Sandra maximises her expected utility according to her beliefs, will she drive or walk to work?

$$EU(Walking) > EU(Driving) \tag{54}$$

$$700.3 > 250 \tag{55}$$

If Sandra acts on her beliefs, we would expect her to walk to work today.

Question 5, b

b) Sandra missed the weather report this morning that stated the true probability of rain today is $4/5$. Given the weather report is accurate, what is Sandra's true expected utility from walking and driving to work?

Question 5, b

$$EU(Walking) = \frac{4}{5} \cdot 1 + (1 - \frac{4}{5}) \cdot 1000 \tag{56}$$

$$EU(Walking) = \frac{4}{5} \cdot 1 + \frac{1}{5} \cdot 1000 \tag{57}$$

$$EU(Walking) = \frac{4}{5} \cdot 1 + \frac{1}{5} \cdot 1000 = 200.8 \tag{58}$$

$$EU(Driving) = 1 \cdot 250 + 0 \cdot 1 \tag{59}$$

Question 5, b

If the weather report is accurate, her expected utility from walking to work is $EU(Walking) = 200.8$. Her expected utility from driving is still 250.

Question 5, b

How much could Sandra increase her expected utility if she listened and believed the weather report?

- Choice under scenario 1, without listening the radio: Walks, $EU = 700.3$
- Choice under scenario 2, listening the weather report: Drives, $EU = 250$
- As she drives she decreases her expected utility by 40.2 units.

Question 6

Sam's utility of wealth function is $U(w) = 15\sqrt{w}$. Sam owns and operates a farm in Dorset. He is concerned that a flood may wipe out his crops. If there is no flood, Sam's wealth is £360,000. The probability of a flood is $1/15$. If a flood does occur, Sam's wealth will fall to £160,000. Calculate the risk premium Sam is willing to pay for full flood insurance.

Question 6

In order to calculate the risk premium, we need to compute the expected utility from facing the two states of nature (flood, no flood):

$$EU(w) = \frac{1}{15} \cdot 15\sqrt{160,000} + (1 - \frac{1}{15}) \cdot 15\sqrt{360,00} \tag{60}$$

$$EU(w) = \frac{1}{15} \cdot 15\sqrt{160,000} + (\frac{14}{15}) \cdot 15\sqrt{360,00} \tag{61}$$

$$EU(w) = 8,800 \tag{62}$$

Question 6

Then, we need to find the level of wealth that Sam needs with certainty to ensure this same level of utility, thus we need to solve for Z:

$$U(Z) = 15\sqrt{Z} = 8,800 \tag{63}$$

$$15^2\sqrt{Z}^2 = 8,800^2 \tag{64}$$

$$225 \cdot Z = 77440000 \Rightarrow Z = \frac{77440000}{225} \tag{65}$$

$$Z = 344,177.76 \tag{66}$$

Sam's risk premium is then the difference between the expected wealth from the lottery and Z.

Question 6

Sam's risk premium is then the difference between his expected level of wealth facing the lottery and and Z.

$$360,000\frac{14}{15} + 160,000\frac{1}{15} = 346,667 \tag{67}$$

$$EMV = 346,667 - 344,177 = 2,489 \tag{68}$$

This implies Sam is willing to pay £2,489 for insurance against a flood.

Microeconomics (5SSPP217)

Seminar

Felipe Torres
felipe.torres@kcl.ac.uk

June 4, 2025

Contact & Office Hours

- My email: **felipe.torres@kcl.ac.uk**
- Office hours: Every Wednesday from 13:00 until 14:00 hrs.
- Happy to meet another day/time.

Question 1, a

1. You must decide how much to study for the final exams of three different subjects. The table below tells you the exam mark you are confident to obtain in each of the exams depending on the number of hours you study.

a) Compute the marginal product of each hour of study for each of the three subjects assuming that without studying your mark would be zero.

Question 1, a

Hours	Mark subject A	Mark subject B	Mark subject C
1	40	60	29
2	65	90	46
3	80	100	60
4	90	100	72
5	95	90	82
6	99	75	90
7	100	55	96
9	100	33	100
9	99	8	100
10	95	0	100

Table: Question 1

Question 1, a

- They are asking us much more marks will increase per an additional hour of study
- They are telling us that without studying, the subjects get zero

The marginal product of input i is the rate-of-change of the output level as the level of input i changes, holding all other input levels fixed

$$MP_i = \frac{\delta y}{\delta x_i} \tag{1}$$

The MP of each hour is different between the mark achieved with or without the last studied.

Question 1, a

For example, for subject A

$$\frac{\partial Mark}{\partial hours} = \frac{\Delta Mark_a}{\Delta Hours_a} = \frac{40-0}{1-0} = 40 \tag{2}$$

For subject B:

$$\frac{\partial Mark}{\partial hours} = \frac{\Delta Mark_b}{\Delta Hours_b} = \frac{60-0}{1-0} = 60 \tag{3}$$

For subject C:

$$\frac{\partial Mark}{\partial hours} = \frac{\Delta Mark_c}{\Delta Hours_c} = \frac{29-0}{1-0} = 29 \tag{4}$$

Question 1, a

For subject A:

Hours	Mark subject A	Mark subject B	Mark subject C
1	40	60	29
2	65	90	46
3	80	100	60
4	90	100	72
5	95	90	82
6	99	75	90
7	100	55	96
9	100	33	100
9	99	8	100
10	95	0	100

$$\frac{40-0}{1-0} = 40$$

$$\frac{65-40}{1-0} = 25$$

$$\frac{80-65}{1-0} = 15$$

$$\frac{90-80}{1-0} = 10$$

$$\frac{99-95}{1-0} = 5$$

$$\frac{100-99}{1-0} = 1$$

$$\frac{100-100}{1-0} = 0$$

Question 1, a

Hours	MPa	MPb	MPc
1	40	60	29
2	25	30	17
3	15	10	14
4	10	0	12
5	5	-10	10
6	4	-15	8
7	1	-20	6
8	0	-22	6
9	-1	-25	0
10	-4	-8	0

Table: Marginal products subjects, Question 1, a

Question 1, b

b) According to the data, does studying have decreasing returns? Why do you think that might be?

Question 1, b

b) According to the data, does studying have decreasing returns? Why do you think that might be?

The law of diminishing marginal returns states that adding an additional unit of a factor of production results in smaller increases in output.

Answer: We see that the MP for all three subjects goes downs as we add one additional hour

Question 1, b

b) E.g. study fatigue or overconfidence

Question 1, c

c) If you value the opportunity cost of each hour of studying at the equivalent to 12 exam marks, how many hours should you study for each exam? What will be your final marks?

Question 1, c

The rule here is that each subjects will keep studying until:

$$MP_i <= 12 \tag{5}$$

Question 1, c

Hours	MPa	MPb	MPc
1	40	60	29
2	25	30	17
3	15	10	14
4	10	0	12
5	5	-10	10
6	4	-15	8
7	1	-20	6
8	0	-22	6
9	-1	-25	0
10	-4	-8	0

Table: Marginal products subjects, Question 1, a

Question 1, c

Answer:

- Subject A: 3 hours $\Rightarrow$ 80
- Subject B: 2 hours $\Rightarrow$ 90
- Subject C: 4 hours $\Rightarrow$ 72

Question 2

2. Show that the perfect substitutes production function $y = a_1x_1 + a_2x_2 + ... + a_nx_n$ and the perfect complements production function $y = \min\{a_1x_1, a_2x_2, ..., a_nx_n\}$ both exhibit constant returns to scale.

Question 2

If, for any input bundle $(x_1, \ldots, x_n)$ for any $k > 0$ then the technology described by the production function f exhibits constant returns-to-scale:

$$f(kx_1, kx_2 \ldots kx_n) = kf(x_1, x_2 \ldots x_n) \tag{6}$$

In other words, if we increase each input by k factor, will give us k times the amount of output.

Question 2

For any $k > 1$ then the technology described by the production function f exhibits diminishing returns-to-scale:

$$f(kx_1, kx_2 ... kx_n) < kf(x_1, x_2 ... x_n) \tag{7}$$

In other words, if we increase each input by k, we get less than k times much output.

For any $k > 1$ then the technology exhibits increasing returns-to-scale:

$$f(kx_1, kx_2 ... kx_n) > kf(x_1, x_2 ... x_n) \tag{8}$$

In other words, if we increase each input by k, we get more than k times as much output.

Question 2

They are asking us if both production functions exhibit constant returns to scale:

$$f(kx_1, kx_2 ... kx_n) = kf(x_1, x_2 ... x_n) \tag{9}$$

Let's multiply each input by k:

$$a_1(kx_1) + a_2(kx_2) + ... + a_n(kx_n) = k(a_1x_1 + a_2x_2 + ... + a_nx_n) \tag{10}$$

For perfect substitutes, we have a common factor k, and we know that $y = a_1x_1 + a_2x_2 + ... + a_nx_n$, thus:

$$k\underbrace{(a_1x_1 + a_2x_2 + ... + a_nx_n)}_{\text{y}} = ky \tag{11}$$

$$ky = ky \tag{12}$$

Question 2

Now let's do the same for **perfect complements**. Let's multiply each input of the function by k, where $k > 0$:

$$\min\{a_1(kx_1), a_2(kx_2)...a_n(kx_n)\} = k\min\{a_1x_1, a_2x_2...a_nx_n\} \tag{13}$$

$$\min\{ka_1(x_1), a_2(x_2)..., a_n(x_n)\} = k\min\{a_1x_1, a_2x_2..., a_nx_n\} \tag{14}$$

$$k\min\{a_1(x_1), a_2(x_2)..., a_n(x_n)\} = k\underbrace{\min\{a_1x_1, a_2x_2..., a_nx_n\}}_{\text{y}} \tag{15}$$

$$k\underbrace{\min\{a_1(x_1), a_2(x_2)..., a_n(x_n)\}}_{\text{y}} = ky \tag{16}$$

$$ky = ky \tag{17}$$

Question 2

A more concrete example to show that taking out the k factor out of the min operator gives you the same results as leaving it inside, let's give any value to x_1 and x_2
If $x_1 = 5, x_2 = \frac{1}{5}$:

$$\min\{10x_1, 10x_2\} = 10\min\{5, \frac{1}{5}\} \tag{18}$$

$$\min\{10 \cdot 5, 10 \cdot \frac{1}{5}\} = 10 \cdot \frac{1}{5} \tag{19}$$

$$\min\{50, 2\} = 2 \tag{20}$$

$$2 = 2 \tag{21}$$

Question 2

Do these production have(or don't have) constant returns to scale?

Question 2

Answer: Yes, both production functions have constant returns to scale.

Question 3

3. Characterise the returns to scale of the following production functions

1. $y = (x_1)^2(x_2)^2$
2. $y = (x_1)^{\frac{1}{4}}(x_2)^{\frac{1}{2}}$
3. $y = 3x_1 + (x_2)^2$
4. $y = (\min\{x_1, x_2\})^{\frac{1}{2}}$

Question 3

$$y = (x_1)^2(x_2)^2 \tag{22}$$

We multiply each input by k ($k > 1$):

$$(kx_1)^2(kx_2)^2 > or < or = k(x_1)^2(x_2)^2 \tag{23}$$

$$k^2x_1^2k^2x_2^2(>,<,=)kx_1^2x_2^2 \tag{24}$$

$$k^{2+2}x_1^2x_2^2 > kx_1^2x_2^2 \tag{25}$$

$$k^4y > ky \tag{26}$$

We know that for any $k > 1$ then the technology exhibits increasing returns-to-scale:

$$f(kx_1, kx_2 ... kx_n) > kf(x_1, x_2 ... x_n) \tag{27}$$

Thus, a firm with this production function exhibits increasing

Question 3

$$y = (x_1)^{\frac{1}{4}}(x_2)^{\frac{1}{2}} \tag{28}$$

$$k((x_1)^{\frac{1}{4}}(x_2)^{\frac{1}{2}}) > or < or = k((x_1)^{\frac{1}{4}}(x_2)^{\frac{1}{2}}) \tag{29}$$

$$(kx_1)^{\frac{1}{4}}(kx_2)^{\frac{1}{2}} > or < or = kx_1^{\frac{1}{4}}x_2^{\frac{1}{2}} \tag{30}$$

$$k^{\frac{1}{4}}x_1^{\frac{1}{4}}k^{\frac{1}{2}}x_2^{\frac{1}{2}} > or < or = kx_1^{\frac{1}{4}}x_2^{\frac{1}{2}} \tag{31}$$

$$k^{\frac{1}{4}+\frac{1}{2}}x_1^{\frac{1}{4}}x_2^{\frac{1}{2}} > or < or = kx_1^{\frac{1}{4}}x_2^{\frac{1}{2}} \tag{32}$$

Question 3

$$k^{\frac{3}{4}} x_1^{\frac{1}{4}} x_2^{\frac{1}{2}} > or < or = k x_1^{\frac{1}{4}} x_2^{\frac{1}{2}} \tag{33}$$

if $k > 1$

$$k^{\frac{3}{4}} y < ky \tag{34}$$

Thus, decreasing returns to scale.

Question 3

$$y = 3x_1 + (x_2)^2 \tag{35}$$

$$3(kx_1) + (kx_2)^2 < \text{or} > \text{or} = k(3x_1 + (x_2)^2) \tag{36}$$

$$3kx_1 + k^2x_2^2 < \text{or} > \text{or} = 3kx_1 + kx_2^2 \tag{37}$$

$$\cancel{3kx_1} + k^2x_2^2 < \text{or} > \text{or} = \cancel{3kx_1} + kx_2^2 \tag{38}$$

If $k > 1$:

$$k^2x_2^2 > kx_2^2 \tag{39}$$

$$f(kx_1, kx_2...kx_n) > kf(x_1, x_2...x_n) \tag{40}$$

Thus, increasing returns to scale.

Question 3

$$y = (\min\{x_1, x_2\})^{\frac{1}{2}} \tag{41}$$

$$(\min\{kx_1, kx_2\})^{\frac{1}{2}} < \text{ or } > \text{ or } = k(\min\{x_1, x_2\})^{\frac{1}{2}} \tag{42}$$

$$(k \min\{x_1, x_2\})^{\frac{1}{2}} < \text{ or } > \text{ or } = k(\min\{x_1, x_2\})^{\frac{1}{2}} \tag{43}$$

$$k^{\frac{1}{2}}(\min\{x_1, x_2\})^{\frac{1}{2}} < \text{ or } > \text{ or } = k(\min\{x_1, x_2\})^{\frac{1}{2}} \tag{44}$$

For $k > 1$:

$$k^{\frac{1}{2}} y < ky \tag{45}$$

$$f(kx_1, kx_2 \ldots kx_n) < kf(x_1, x_2 \ldots x_n) \tag{46}$$

Thus, decreasing returns to scale.

Question 4

Sunrise Ltd. employs only labour hours to produce widgets. The firm is a price taker both in the labour market and in the market of widgets. Denote by w the hour wage and by p the price of widgets. The production function of Sunrise is $y = AL$ where $A > 0$. **Find the optimal input demand and optimal input supply of the firm.**

Question 4

We can derive the firm's profit function with respect to our factor of production L:

The optimal level of input of L is where the $pMP_L = w_1$, we can obtain this by deriving

$$\pi(L) = pAL - wL \tag{47}$$

$$\frac{\partial \pi(L)}{\partial L} = \frac{\partial pAL}{\partial L} - \frac{\partial wL}{\partial L} \tag{48}$$

$$\frac{\partial \pi(L)}{\partial L} = pA - w = 0 \tag{49}$$

Question 4

If $pA = w$, then $p = \frac{w}{A}$
Then, the firm get zero profits, and it will be indifferent on the amount of L.

$$\pi(L) = \frac{W}{\cancel{A}}\cancel{A}L - wL \Rightarrow wL - WL \Rightarrow 0 \tag{50}$$

$$L^* = (0, \infty_+) \tag{51}$$

Optimal supply of input $y^* = (0, \infty_+)$. We knew that the profit maximisation problem only has a solution under a constant returns to scale technology when profits are zero.

Question 4

If $w < pA$:

$$\frac{\partial \pi(L)}{\partial L} = pA - w > 0 \tag{52}$$

$$\frac{\partial \pi(L)}{\partial L} > 0 \Rightarrow L^* = \infty_+ \tag{53}$$

Optimal supply of input: $y^* = \infty_+$

Question 4

Conversely, if $w > pA$

$$\frac{\partial \pi(L)}{\partial L} = pA - w < 0 \tag{54}$$

$$\frac{\partial \pi(L)}{\partial L} < 0 \Rightarrow L^* = 0 \text{ Optimal input demand} \tag{55}$$

Optimal supply of input: $y^* = 0$

Question 5

Acme Container Co. produces egg cartons that are sold to egg distributors. Acme has estimated this production function for its egg carton division: $y = 25L^{0.6}K^{0.3}$, where y is the output measured in one thousand carton lots, L is labour measured in person hours, and K is capital measured in machine hours. Acme currently pays a wage of £10 per hour and considers the relevant rental price for capital to be £25 per hour. The selling price of egg cartons is p.

Question 5, a

a) Determine the capital-labour ratio that Acme should use in the egg carton division in order to maximise its profits.

Question 5, a

We know that firms maximise their profits when the marginal rate of technical substitution is equal to the ratio of the prices of these inputs:

$$MTRS = \frac{\frac{\partial y}{\partial x_1}}{\frac{\partial y}{x_2}} = \frac{w_1}{w_2} \tag{56}$$

The profit-maximising level of each input is:

$$p \cdot MP_1 - w_1 = 0 \text{ and } p \cdot MP_2 - w_2 = 0 \Rightarrow MTRS = \frac{w_1}{w_2} \tag{57}$$

Question 5, a

Let's calculate first the MP of L:

$$y = 25L^{0.6}K^{0.3} \Rightarrow \frac{\partial y}{\partial L} = MP_L \tag{58}$$

$$MP_L = \frac{\partial 25L^{0.6}K^{0.3}}{\partial L} \tag{59}$$

$$MP_L = 0.6 \cdot 25L^{0.6-1}K^{0.3} \Rightarrow 15\frac{K^{0.3}}{L^{0.4}} \tag{60}$$

$$MP_L = 15\frac{K^{0.3}}{L^{0.4}} \tag{61}$$

Question 5, a

Let's calculate first the MP of K:

$$y = 25L^{0.6}K^{0.3} \Rightarrow \frac{\partial y}{\partial K} = MP_K \tag{62}$$

$$MP_K = \frac{\partial 25L^{0.6}K^{0.3}}{\partial K} \tag{63}$$

$$MP_K = 0.3 \cdot 25L^{0.6}K^{0.3-1} \Rightarrow 7.5\frac{L^{0.6}}{K^{0.7}} \tag{64}$$

$$MP_K = 7.5\frac{L^{0.6}}{K^{0.7}} \tag{65}$$

Question 5, a

Now, let's calculate the MTRS:

$$MTRS = \frac{MP_K}{MP_L} \tag{66}$$

$$MTRS = \frac{7.5\frac{L^{0.6}}{K^{0.3}}}{15\frac{K^{0.3}}{L^{0.7}}} \Rightarrow \frac{\cancel{7.5}L^{0.4} \cdot L^{0.6}}{\cancel{15}K^{0.3} \cdot K^{0.7}} \Rightarrow \frac{L}{2K} \tag{67}$$

$$MTRS = \frac{L}{2K} \tag{68}$$

Question 5, a

We know that $w = 10$ and $r = 25$

$$MTRS = \frac{r}{w} \tag{69}$$

$$\frac{L}{2K} = \frac{25}{10} \Rightarrow \frac{K}{L} = \frac{\cancel{10}^{1}}{\cancel{50}^{5}} \tag{70}$$

$$\frac{K}{L} = \frac{1}{5} = 0.2 \tag{71}$$

Question 5, b

b) Write the profit function of the firm and find analytically its optimal choices of capital and labour K* and L* as a function of output price p

Question 5, b

Our profit function is:

$$\pi(K, L, w, r) = 25pL^{0.6}K^{0.3} - 10L - 25K \tag{72}$$

Question 5, b

Our profit function is:

$$\pi(K, L, w, r) = \underbrace{25pL^{0.6}K^{0.3}}_{\text{Total Revenue}} - \underbrace{10L - 25K}_{\text{Total Cost}} \tag{73}$$

Question 5, b

Our profit function is:

$$\pi(K, L, w, r) = 25L^{0.6}K^{0.3} - 10L - 25K \tag{74}$$

We know that a firm maximises its utility when:

$$p \cdot MP_1 - w_1 = 0 \text{ and } p \cdot MP_2 - w_2 = 0 \tag{75}$$

Thus, we can derive our profit function wrt to L and K and equate them to the market value of K and L:

$$\frac{\partial \pi}{\partial L} = \frac{p15K^{0.3}}{L^{0.4}} = 10 \tag{76}$$

$$\frac{\partial \pi}{\partial K} = \frac{p7.5L^{0.6}}{K^{0.7}} = 25 \tag{77}$$

Question 5, b

$$\frac{\partial \pi}{\partial L} = \frac{p15K^{0.3}}{L^{0.4}} = 10 \tag{78}$$

$$\frac{\partial \pi}{\partial K} = \frac{p7.5L^{0.6}}{K^{0.7}} = 25 \tag{79}$$

We also know in equilibrium:

$$\frac{K}{L} = 0.2 \tag{80}$$

Question 5, b

We can arrange K as a function of L:

$$\frac{K}{L} = 0.2 \Rightarrow K = 0.2L \tag{81}$$

$$\frac{\partial \pi}{\partial L} = \frac{p15K^{0.3}}{L^{0.4}} = 10 \tag{82}$$

$$\frac{\partial \pi}{\partial L} = \frac{p15(0.2L)^{0.3}}{L^{0.4}} = 10 \tag{83}$$

$$\frac{\partial \pi}{\partial L} = \frac{p15(0.2^{0.3}L^{0.3})}{L^{0.4}} = 10 \tag{84}$$

$$L^{-0.1} = \frac{1.079}{p} \tag{85}$$

$$L^{*} = 0.461p^{10} \Rightarrow K^{*} = 0.092p^{10} \tag{86}$$

Question 6

6. Consider a firm with a technology represented by production function $y = \min\{x_1, \sqrt{x_2}\}$. Prices of inputs are w_1 and w_2 and the price of the output the firm sells is p. Which type of returns to scale does this function exhibits? Find the optimal choice of inputs x_1^* and x_2^* and the resulting output as a function of the input prices w_1 and w_2 and the output price p.

Question 6

Which type of returns to scale does this function exhibits?

Question 6

We multiply each input by k:

$$\min\{kx_1, \sqrt{kx_2}\} >, < \text{ or } = k\min\{x_1, \sqrt{x_2}\} \tag{87}$$

$$\min\{kx_1, \sqrt{k}\sqrt{x_2}\} = \text{ or } > \text{ or } < k\min\{x_1, \sqrt{x_2}\} \tag{88}$$

We multiply and divide by k $\sqrt{x_2}$, which is equal to multiply by 1 x_2:

$$\min\{kx_1, \frac{k\sqrt{k}\sqrt{x_2}}{k}\} = \text{ or } > \text{ or } < k\min\{x_1, \sqrt{x_2}\} \tag{89}$$

Question 6

We take out k out of the min operator:

$$k \min\{x_1, \frac{\sqrt{k}\sqrt{x_2}}{k}\} = \text{or} > \text{or} < k \min\{x_1, \sqrt{x_2}\} \tag{90}$$

$$k \min\{x_1, k^{\frac{1}{2}-1}\sqrt{x_2}\} = \text{or} > \text{or} < k \min\{x_1, \sqrt{x_2}\} \tag{91}$$

$$k \min\{x_1, \frac{\sqrt{x_2}}{\sqrt{k}}\} < k \min\{x_1, \sqrt{x_2}\} \tag{92}$$

Here we are now dividing our production by $\sqrt{k}$, thus decreasing returns to scale.

Question 6

Find the optimal choice of inputs x_1^* and x_2^* and the resulting output as a function of the input prices w_1 and w_2

Question 6

Can we obtain the FOCs to obtain the optimal choices of inputs given this production function?

Question 6

- This production function is not differentiable, so we cannot just use derivatives. We need to think economically in terms of revenue and costs.
- However, the conditions to find the optimal level of production for long-run profit maximisation do not change given a particular production function.
- It's always when $pxMP_n$ is the marginal revenue product of input n is equal to $pxMP_n = w_n$
- That is, marginal revenue of all inputs equals the marginal cost for all inputs

Question 6

Hence, the marginal revenue product for input x_2 is equal to $pMP_2 = w_2$

$$y = \sqrt{x_2} \tag{93}$$

$$MP_2 = \frac{\partial y}{\partial x_2} = \frac{1}{2\sqrt{x_2}} \tag{94}$$

$$p \cdot MP_2 = \frac{p}{2\sqrt{x_2}} \tag{95}$$

Note that the marginal revenue product of input 2 is $\frac{p}{2\sqrt{x_2}}$ whilst the price of input is w_2.

Question 6

This means that at any optimal choice the two must be equal:

$$\frac{p}{2\sqrt{x_2}} = w_2 \tag{96}$$

$$p = w_2 \cdot 2\sqrt{x_2} \tag{97}$$

$$\sqrt{x_2} = \frac{p}{2w_2} \tag{98}$$

$$x_2^* = \frac{p^2}{4w_2^2} \tag{99}$$

Question 6

Now, note that whatever the optimal choice is, it must also meet the following condition: $x_1 = \sqrt{x_2}$, Otherwise the firm is buying inputs which do not contribute to increase output. Hence:

$$x_1 = \sqrt{x_2} \tag{100}$$

$$x_1 = \sqrt{\frac{p^2}{4w_2^2}} \tag{101}$$

$$x_1^* = \frac{p}{2w_2} \tag{102}$$

Question 7, a

7. Consider a firm whose technology is represented by the production function $y = 3x1 + x2$. Prices of inputs are w_1 and w_2 and the price of the output the firm sells is p.

a) In the short run, the quantity of input x_2 is fixed at 2. Write the short run profit function for the firm. Find the short run optimal choice of input x_1^* as a function of the ratio w1/p.

Question 7, a

Write the short run profit function for the firm:

$$\pi = \text{revenue} - \text{costs} \tag{103}$$

$$\pi = p \cdot y - \text{costs} \tag{104}$$

$$\pi = p \cdot (3x_1 + x_2) - w_1 x_1 - x_2 w_2 \tag{105}$$

They tell us that $x_2 = 2$:

$$\pi = p \cdot (3x_1 + 2) - w_1 x_1 - 2w_2 \tag{106}$$

Question 7, a

Find the short run optimal choice of input x_1^* as a function of the ratio w1/p:

Question 7, a

We know that a firm will maximise its short-term profit when the marginal revenue product is equal to the price of that input, but that's the same as the derivative of the profit function the w.r.t to x_1

$$\pi(x_1, \tilde{x_2}, w_1, w_2) = p \cdot (3x_1 + 2) - w_1 x_1 - 2w_2 \tag{107}$$

$$p \cdot MP_1 = w_1 \tag{108}$$

This is equal to:

$$\frac{\partial \pi}{\partial x_1} = p \cdot \frac{\partial y}{\partial x_1} - w_1 = 0 \tag{109}$$

$$\frac{\partial \pi}{\partial x_1} = 3p - w_1 = 0 \tag{110}$$

Question 7, a

Then, the optional choice of input 1 depends on the relationship between p and w_1:

$$p = \frac{w_1}{3} \tag{111}$$

- If $p = \frac{w_1}{3}$
- The profit is zero, and the firm is indifferent about buying any amount of input 1

Question 7, a

Then, the optional choice of input 1 depends on the relationship between p and w_1:

- If $p > \frac{w_1}{3}$
- The profit function is increasing everywhere.
- In other words, the marginal revenue of input 1 is greater than its marginal cost
- Hence the firm would like to buy infinite amounts of input 1
- $x_1^* = \infty^+$

Question 7, a

Then, the optional choice of input 1 depends on the relationship between p and w_1:

- The opposite holds for $p < \frac{w_1}{3}$
- In this case the firm would make losses by buying any positive amount of input 1, so the firm would make losses by buying any positive amount of input 1
- $x_1^* = 0$

Question 7, b

b) Consider now the long-run. Find the optimal choice of inputs x_1^* and x_2^* and the resulting output as a function of the input prices w_1 and w_2 and the output price p.

Question 7, b

In order to solve for this long-run profit maximisation problem, we need to first set up the profit function.

$$\pi(x_1, x_2, w_1, w_2) = p \cdot (3x_1 + x_2) - w_1 x_1 - w_2 x_2 \tag{112}$$

We then obtain the FOCs with wrt to x_1 and x_2:

$$\frac{\partial \pi(x_1, x_2, w_1, w_2)}{\partial x_1} = \frac{\partial p \cdot (3x_1 + x_2) - w_1 x_1 - w_2 x_2}{\partial x_1} = 0 \tag{113}$$

$$\frac{\partial \pi(x_1, x_2, w_1, w_2)}{\partial x_2} = \frac{\partial p \cdot (3x_1 + x_2) - w_1 x_1 - w_2 x_2}{\partial x_2} = 0 \tag{114}$$

Question 7, b

$$\frac{\partial \pi(x_1, x_2, w_1, w_2)}{\partial x_1} = 3p - w_1 = 0 \tag{115}$$

$$\frac{\partial \pi(x_1, x_2, w_1, w_2)}{\partial x_2} = p - w_2 = 0 \tag{116}$$

Question 7, b

$$3p = w_1 \Rightarrow p = \frac{w_1}{3} \tag{117}$$

$$p = w_2 \tag{118}$$

Question 7, b

Conclusion:

$p = \frac{w_1}{3} = w_2$, then:

The firm makes zero profit whatever its choice of inputs, so the firm is willing to choose any combination of inputs.

Question 7, b

$$\text{If } p > \frac{w_1}{3} \tag{119}$$

- The firm maximises its profits by buying as much input 1 as possible

$$\text{If } p < \frac{w_1}{3} \tag{120}$$

- The firm would make a loss by buying input 1

Question 7, b

$$\text{If } p > w_2 \tag{121}$$

- The firm maximises its profits by buying as much input 2 as possible

$$\text{If } p < w_2 \tag{122}$$

- The firm would make a loss by buying input 2

Question 7, b

Hence, the optimal choice of input is the following:

$p > \max\{w_1/3, w_2\}$, then:

$$\text{Infinite amounts of } x_1 \text{ and } x_2 \text{ if } p > \max\{w_1/3, w_2\}$$

$w_2 > p > \frac{w_1}{3}$, then:

$$\text{Infinite amounts of } x_1 \text{ and zero of } x_2$$

$p < \min\{\frac{w_1}{3}, w_2\}$, then:

$$\text{Zero amounts of } x_1 \text{ and zero of } x_2$$

Microeconomics (5SSPP217)

Seminar

Felipe Torres
felipe.torres@kcl.ac.uk

June 4, 2025

Contact & Office Hours

- My email: **felipe.torres@kcl.ac.uk**
- Fridays between 11:00 to 12:00 hrs. Bush House (NE). 9.01

Question 1

1. Find analytically the conditional factor demand for the two inputs

Question 1

1. Firm A has the following technology: $f(x_1; x_2) = \sqrt{(x_1 x_2)}$. The price of input 1 is $w_1 = 8$ and the price of input 2 is $w_2 = 2$. The Firm A wants to produce y units of output. Find analytically the conditional factor demand for the two inputs. Draw an isoquant a several isoprofit lines. Locate the cost-minimising input bundle. Explain the intuition.

Question 1

1. Find analytically the conditional factor demand for the two inputs

Question 1

Locate the cost-minimising input bundle. We have a production function and input prices, in order to find the conditional factor demand of the inputs, we need to solve for this cost-minimisation problem, subject to a level of production. Stated formally:

$$\min w_1 x_1 + w_2 x_2 \tag{1}$$

$$x_1, x_2 \geq 0$$

$$\text{s.t:} \;\; y = \sqrt{(x_1 x_2)}$$

Then, we do the same that is on slide 22, lecture 9, thus we set up the Lagrangian:

$$L(x_1, x_2, \lambda) = w_1 x_1 + w_2 x_2 - \lambda(\sqrt{(x_1 x_2)} - y) \tag{2}$$

Question 1

Then, we obtain the FOCs wrt x_1, x_2, λ

$$L(x_1, x_2, \lambda) = 8x_1 + 2x_2 - \lambda(\sqrt{(x_1 x_2)} - y) \tag{3}$$

$$\frac{\partial L}{\partial x_1} = 8 - \frac{\lambda\sqrt{x_2}}{2\sqrt{x_1}} = 0 \Longrightarrow \sqrt{x_1} = \frac{\lambda\sqrt{x_2}}{16} \tag{4}$$

$$\frac{\partial L}{\partial x_2} = 2 - \frac{\lambda\sqrt{x_1}}{2\sqrt{x_2}} = 0 \longrightarrow \lambda = \frac{4\sqrt{x_2}}{\sqrt{x_1}} \tag{5}$$

$$\frac{\partial L}{\partial \lambda} = -\sqrt{(x_1 x_2)} + y = 0 \tag{6}$$

Question 1

If you plug in the λ into the first FOC, you will get:

$$\sqrt{x_1} = \frac{\lambda\sqrt{x_2}}{16} \tag{7}$$

$$\sqrt{x_1} = \frac{\frac{4\sqrt{x_2}\sqrt{x_2}}{\sqrt{x_1}}}{16} \implies \sqrt{x_1} = \frac{\frac{4x_2}{\sqrt{x_1}}}{16} \implies \sqrt{x_1} = \frac{4x_2}{\sqrt{x_1}16} \tag{8}$$

$$4\sqrt{x_1}\sqrt{x_1} = x_2 \tag{9}$$

$$4x_1 = x_2 \tag{10}$$

Question 1

Alternatively, we could set the TRS equal to the input price ratio:

$$\frac{w_1}{w_2} = \frac{MP_1}{MP_2} = TRS \tag{11}$$

$$\frac{8}{2} = \frac{\frac{\sqrt{x_2}}{2\sqrt{x_1}}}{\frac{\sqrt{x_1}}{2\sqrt{x_2}}} \implies 4 = \frac{\frac{\sqrt{x_2}}{\cancel{2}\sqrt{x_1}}}{\frac{\sqrt{x_1}}{\cancel{2}\sqrt{x_2}}} \tag{12}$$

$$4 = \frac{\sqrt{x_2}\sqrt{x_2}}{\sqrt{x_1}\sqrt{x_1}} \implies 4 = \frac{x_2}{x_1} \tag{13}$$

$$x_2 = 4x_1 \tag{14}$$

Question 1

If you plug x_2 as a function of x_1 into the production function, you will get the conditional factor demand for input 1:

$$y = \sqrt{(x_1 x_2)} \tag{15}$$

$$x_2 = 4x_1 \tag{16}$$

$$y = \sqrt{(x_1 \cdot 4x_1)} \Longrightarrow y = 2x_1 \tag{17}$$

$$x_1^* = \frac{y}{2} \tag{18}$$

This is the firm's conditional demand for input 1.

Question 1

The second step is to solve x_2

$$y = \sqrt{(x_1 x_2)} \tag{19}$$

$$x_2 = 4x_1 \longrightarrow x_1 = \frac{x_2}{4} \tag{20}$$

$$y = \sqrt{(\frac{x_2}{4} \cdot x_2)} \Longrightarrow y = \frac{x_2}{2} \tag{21}$$

$$x_2^* = 2y \tag{22}$$

This is the firm's conditional demand for input 2.

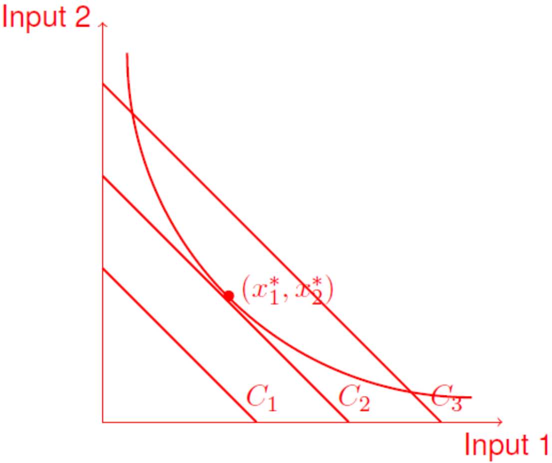
Question 1

Let's think about the intuition:

The intuition behind the optimality condition is that, at the optimum, the relative cost of inputs must equate the rate at which the firm can substitute one input for the other whilst keeping the output constant. Otherwise the firm is not minimising cost.

Question 1

Draw an isoquant a several isoprofit lines.



Question 1

In order to draw the isocost: Set up x_2 as a function of x_1, holding y constant, then changing y and computing x_2 for different values of x_1.

$$c = 8x_1 + 2x_2 \implies 2x_2 = c - 8x_1 \implies x_2 = \frac{c}{2} - 4x_1 \tag{23}$$

We can see that it is a linear function. with slope -4 and intercepts x_2 at $\frac{y}{2}$

We do the same for the isoquant. You can fix y to a certain value and try different values of x_1.

$$y = \sqrt{x_1 x_2} \implies \sqrt{x_2} = \frac{y}{\sqrt{x_1}} \implies x_2 = \frac{y^2}{x_1} \tag{24}$$

Question 2, a)

2. A cost-minimising firm has a production function $y = \min\{4x_1, x_2\}$. Input prices are w1 and w2

a) For a given target output Y, draw the corresponding isoquant and a number of isocost lines. Identify in the graph the cost minimising input bundle.

Question 2, a)

Let's start with the isocost lines:

$$w_1 x_1 + w_2 x_2 = c \tag{25}$$

$$x_2 = -\frac{w_1 x_1}{w_2} + \frac{c}{w_2} \tag{26}$$

Question 2, a)

We see that this is a linear function with slope $\frac{-w_1}{w_2}$ and intercept $\frac{c}{w_2}$

$$x_2 = \frac{-w_1 x_1}{w_2} + \frac{c}{w_2} \tag{27}$$

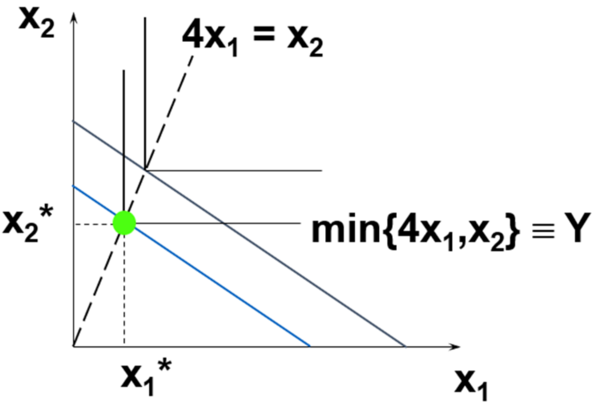
Question 2, a)

For this production function with fixed proportions, the isoquant is a L-shaped curve with a kink at $x_2 = 4x_1$

However, on this fixed proportion production function, models a technology in which $\frac{1}{4}$ units of input x_1 and 1 unit of input 2 are required to produce every unit of output. (Not perfect complements).

Question 2, a)

a) Let's draw now the isoquant and isocost:



Question 2, b)

b) Find analytically the cost minimising bundle and the resulting cost function.

Question 2.b

We know that we attain the optimal demand of both inputs for these types of functions when $x_1 = x_2$, which in this case $4x_1 = x_2$

$$y = 4x_1 \longrightarrow x_1^* = \frac{y}{4} \tag{28}$$

$$y = x_2 \longrightarrow x_2^* = y \tag{29}$$

Question 2, b)

We also know that this firm's total cost minimisation function is our function, but plugging in the conditional factor demands:

$$C(w_1, w_2, y) = w_1 x_1 * (w_1, w_2, y) + w_2 x_2 * (w_1, w_2, y) + \tag{30}$$

$$C(w_1, w_2, y) = w_1 \frac{y}{4} + w_2 y \tag{31}$$

$$C(w_1, w_2, y) = (\frac{w_1}{4} + w_2) y \tag{32}$$

Question 3, a)

3) A cost-minimising firm has a production function $y = L + 2K$. Input prices are w_L for labour and w_K for capital.

a) For a given target output Y, draw the corresponding isoquant and several isocost lines with different slopes. Identify in the graph the cost minimising input bundle in each case.

Question 3, a)

a)Draw the corresponding isoquant and several isocost lines with different slopes.

$$x_2 = -\frac{w_1}{w_2} + \frac{c}{w_2} \tag{33}$$

In this exercise, we get the following equation, with the following slope:

$$K = -\frac{w_k L}{w_L} + \frac{c}{w_K} \tag{34}$$

$$\text{Slope} = -\frac{w_K}{w_L} \tag{35}$$

Question 3, a)

a) Let's draw now the isoquant:

$$y = L + 2K \tag{36}$$

a) We see that the slope here is equal to $\frac{-1}{2}$:

$$-2K = L - y \implies -K = \frac{L}{2} - \frac{y}{2} \tag{37}$$

$$K = \frac{y}{2} - \frac{L}{2} \tag{38}$$

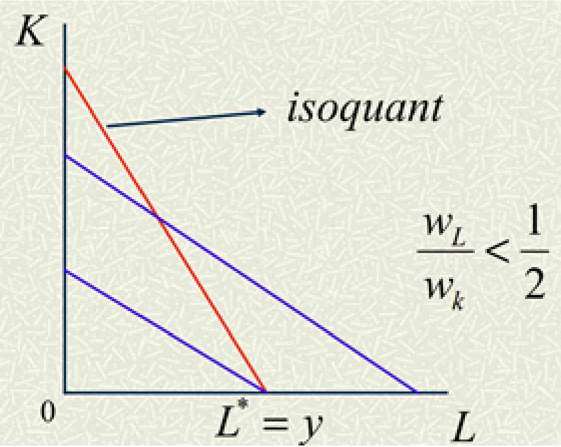
Thus, the slope of this isoquant is $\frac{-1}{2}$:

Question 3, a)

a) Identify in the graph the cost minimising input bundle.

Question 3, a)

a) Let's draw now the isoquants:



Question 3, a)

Scenario 1:

$$\frac{w_L}{w_K} < 1/2 \tag{39}$$

The **isocost** is flatter than the **isoquant** and the cost minimising bundle is located at the horizontal axis and the firm only uses labour.

Scenario 2:

$$\frac{w_L}{w_K} > 1/2 \tag{40}$$

The **isoquant** is flatter than the **isocost** and the cost minimising bundle is located at the vertical axis and the firm only uses capital.

Question 3, b)

b) Find analytically the cost minimising bundle and the resulting cost function as a function of the ratio $\frac{w_L}{w_K}$.

Question 3, b)

Let's find the optimal demand functions for the different scenarios:

$$\frac{w_l}{w_k} = \frac{1}{2} \tag{41}$$

Any combination that produces the desired Y units of output has the same total costs.

Question 3, b)

Let's find the optimal demand functions for the different scenarios:

$$K^* = \begin{cases} 0 \text{ if } \frac{w_L}{w_K} < \frac{1}{2} \\ y/2 \text{ if } \frac{w_L}{w_K} > \frac{1}{2} \end{cases} \tag{42}$$

$$L^* = \begin{cases} y \text{ if } \frac{w_L}{w_K} < \frac{1}{2} \\ 0 \text{ if } \frac{w_L}{w_K} > \frac{1}{2} \end{cases} \tag{43}$$

$\{K^* = 0, L^* = y\}$ or $\{K^* = \frac{y}{2}, L^* = 0\}$

The intuition is that the relative price of labour in the case that $\frac{w_L}{w_K} < 1/2$ is always smaller than its relative marginal productivity (RTS), so the firm should only buy labour.

Question 3, b)

$\{K^* = 0, L^* = y\}$ or $\{K^* = \frac{y}{2}, L^* = 0\}$

$$C(w_L, w_K, y) = \begin{cases} w_k \cdot 0 + w_L \cdot y \text{ if } \frac{w_L}{w_K} < \frac{1}{2} \\ w_K \cdot \frac{y}{2} + w_L \cdot 0 \text{ if } \frac{w_L}{w_K} > \frac{1}{2} \end{cases} \tag{44}$$

The resulting cost functions are then:

$$C(w_L, w_K, y) = \begin{cases} w_L \cdot y \text{ if } \frac{w_L}{w_K} < \frac{1}{2} \\ w_K \cdot \frac{y}{2} \text{ if } \frac{w_L}{w_K} > \frac{1}{2} \end{cases} \tag{45}$$

Question 4, a)

A firm's cost function is given by $c(y) = 4000 + 5y + 10y^2$

a) Write an expression for each of the following cost concepts and draw the resulting curves:

1. Total Fixed Cost
2. Average Fixed Cost
3. Total Variable Cost
4. Average Variable Cost
5. Average Total Cost
6. Marginal Cost

Question 4, a)

a) Let's break down this production function:

$$c(y) = \underbrace{4000}_{\text{Fixed cost}} + \underbrace{5y + 10y^2}_{\text{Variable cost}} \tag{46}$$

- Thus, $F = 4000$ is the Total Fixed Cost (TFC)
- $5y + 10y^2$ is the Total Variable Cost(TVC).

Question 4, a)

a) Write an expression for each of the following cost concepts and draw the resulting curves. Let's review the formulas

$c(y) = F + c_v(y)$

1. Total Fixed Cost F
2. Average Fixed Cost $AFC(y, F) = \frac{F}{y}$
3. Total Variable Cost $TVC(y) = c_v(y)$
4. Average Variable Cost $AVC(y) = \frac{c_v(y)}{y}$
5. Average Total Cost $ATC(y, F) = \frac{F}{y} + \frac{c_v(y)}{y}$
6. Marginal Cost $MC(y) = \frac{\partial c(y)}{\partial y}$

Question 4, a)

a) Then, the Average Fixed Cost (AFC) is:

$$AFC(F, y) = \frac{F}{y} = \frac{4000}{y} \tag{47}$$

The Average Variable Cost (AVC) is equal to:

$$AVC(y) = \frac{c_v(y)}{y} = \frac{5y}{y} + \frac{10y^2}{y} \Longrightarrow 5 + 10y \tag{48}$$

Question 4, a)

a) Finally, the Average Total Cost (ATC(y,F))

$$ATC(y,F) = \frac{F}{y} + \frac{c_v(y)}{y} \quad (49)$$

$$ATC(y,F) = \frac{4000}{y} + \frac{5y}{y} + \frac{10y^2}{y} \quad (50)$$

$$ATC(y,F) = \frac{4000}{y} + 5 + 10y \quad (51)$$

Question 4, a)

a) Finally, and the Marginal Cost are:

$$MC(y) = \frac{\partial C_V(y)}{\partial y} \tag{52}$$

$$c(y) = 4000 + 5y + 10y^2 \tag{53}$$

$$MC(y) = \frac{\partial c(y)}{\partial y} = \frac{\partial 5y}{\partial y} + \frac{\partial 10y^2}{\partial y} \tag{54}$$

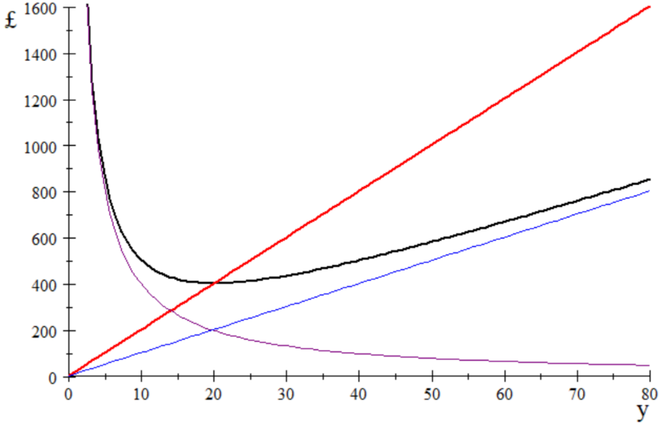
$$MC(y) = 5 + 2 \cdot 10y^{2-1} \tag{55}$$

$$MC(y) = 5 + 20y \tag{56}$$

Question 4, a)

a) Draw the resulting curves:

–AFC –AVC –ATC –MC



Question 4, b)

b) Find analytically the quantity that minimises the average total cost. Demonstrate that the marginal cost is higher (lower) than the average cost for output levels above (below) that quantity.

Question 4, b)

First, let's remember that:

$$c(y) = F + c_v(y) \tag{57}$$

$$MC(y) = \frac{\partial c(y)}{\partial y} = \frac{\partial c_v}{\partial y} \tag{58}$$

Slides 38 to 45, "The short-run MC curves intersects the short-run AVC (or ATC) curve from below at the AVC curve's (ATC curve's) minimum:

$$MC(y) = ATC(y) \Longrightarrow \frac{\partial ATC(y)}{\partial y} = 0 \tag{59}$$

Question 4, b)

We have our ATC function:

$$ATC(y, F) = \frac{4000}{y} + 5 + 10y \tag{60}$$

$$\frac{\partial ATC(y, F)}{\partial y} = \frac{\partial 4000y^{-1}}{\partial y} + \frac{\partial 5}{\partial y} + \frac{\partial 10y}{\partial y} = 0 \tag{61}$$

$$-1 \cdot 4000y^{-2} + 10 = 0 \tag{62}$$

$$4000y^{-2} = 10 \implies y^{-2} = \frac{10}{4000} \implies y^2 = 400 \tag{63}$$

$$y = 20 \tag{64}$$

Question 4, b)

Let's check that the MC and the ATC are the same at y = 20:

$$ATC = \frac{4000}{y} + 5 + 10y \tag{65}$$

$$ATC = \frac{4000}{20} + 5 + 10 \cdot 20 \longrightarrow 200 + 5 + 200 = 405 \tag{66}$$

Now, let's check the MC:

$$MC = 5 + 20y \tag{67}$$

$$MC = 5 + 20 \cdot 20 \longrightarrow 5 + 400 = 405 \tag{68}$$

Question 4, b)

b) Demonstrate that the marginal cost is higher(lower) than the average cost for output levels above(below) that quantity:

We can get the second derivative of the ATC to find out whether is decreasing or increasing:

$$ATC = \frac{4000}{y} + 5 + 10y \tag{69}$$

First derivative:

$$\frac{1}{y^2} - 400 = 0 \longrightarrow y = 20 \tag{70}$$

Second derivative:

$$\frac{-1}{2y^3} = 0 \tag{71}$$

For any value of $y > 0$, the second derivative is negative, thus is a minimum.

Question 4, b)

$$\text{if } MC < ATC \longrightarrow \text{then } 5 + 20y < \frac{4000}{y} + 5 + 10y \qquad (72)$$

$$\text{if } MC < ATC \longrightarrow \text{then } 5 - 5 + 20y - 10 < \frac{4000}{y} \qquad (73)$$

$$\text{if } MC < ATC \longrightarrow \text{then } 10y < \frac{4000}{y} \qquad (74)$$

$$\text{if } MC < ATC \longrightarrow \text{then } y \cdot y < \frac{4000}{10} \qquad (75)$$

Question 4, b)

$$\text{if } MC < ATC \longrightarrow \text{then } y^2 < 400 \tag{76}$$

$$\text{if } MC < ATC \longrightarrow \text{then } y < 20 \tag{77}$$

$$\text{if } MC > ATC, y > 20 \tag{78}$$

Question 5, a)

5. Domination Pizzas' short-run cost function is $C(y,K) = \frac{17y^2}{32K} + 0.25K$, where y is the number of pizzas produced and K is the number of ovens it uses. Currently, Domination Pizzas is leasing 4 ovens in the short run.

a) Calculate the average cost of producing 10 pizzas.

Question 5, a)

5.a) Calculate the average cost of producing 10 pizzas.

$$c(y,K) = \frac{17y^2}{32k} + 0.25K \tag{79}$$

$$ATC(y,K) = \frac{c(y,K)}{y} \implies \frac{\frac{17y^2}{32K} + 0.25K}{y} \tag{80}$$

$$ATC(10,4) = \frac{\frac{17 \cdot 10^2}{32 \cdot 4} + 0.25 \cdot 4}{10} \tag{81}$$

$$ATC(10,4) = 1.43 \tag{82}$$

Question 5, b)

b) The manager of Domination is considering leasing 10 additional ovens. What would be then the average total cost of producing 10 pizzas? Would it be higher or lower than the average cost with 4 ovens? Why?

Question 5, b)

$$ATC(y, K) = \frac{c(y, k)}{y} \Longrightarrow \frac{\frac{17y^2}{32K} + 0.25K}{y} \tag{83}$$

$$ATC(10, 14) = \frac{\frac{17 \cdot 10^2}{32 \cdot 14} + 0.25 \cdot 14}{10} \tag{84}$$

$$ATC(10, 14) = 0.72 \tag{85}$$

Adding 10 ovens will decrease the average cost of producing 10 pizzas. The reason is that 14 ovens is closer to the long run minimum cost of producing 10 pizzas.

Question 5, b)

We can show this by finding k when y = 10:

$$\frac{\partial ATC(y,k)}{\partial y} = \frac{34y - 1 \cdot \frac{17y^2}{32K} + 0.25k}{y^2} = 0 \tag{86}$$

$$\frac{\partial ATC(y,k)}{\partial y} = \frac{34 \cdot 10 - 1 \cdot \frac{17 \cdot 10^2}{32K} + 0.25k}{10^2} = 0 \tag{87}$$

$$K^* = 14.57 \tag{88}$$

Domination was using too few ovens so getting closer to the long run optimum reduces the ATC of producing the 10 pizzas.

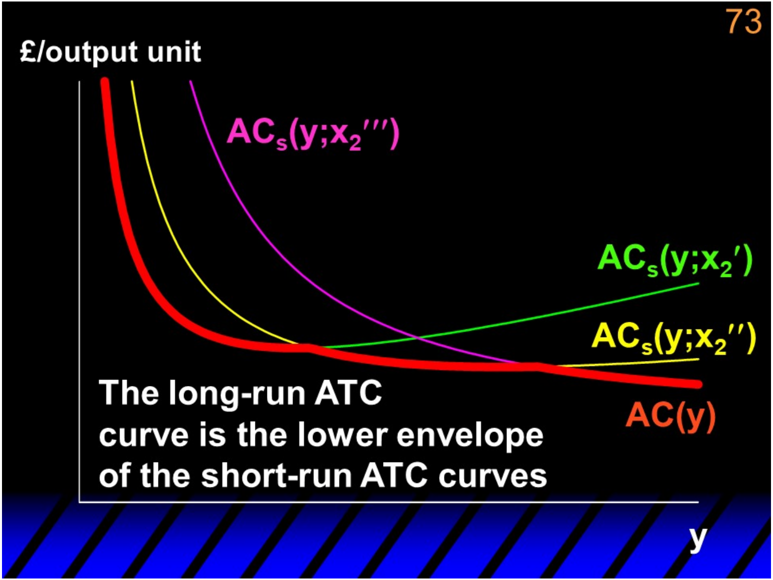
Question 5, c)

c) Find Domination's long run cost function. Does it exhibit increasing, constant, or decreasing returns to scale?

Question 5, c)

We know that the long run cost function and the short run function coincide when the amount of input fixed in the short run coincides with the amount of that input that minimises cost in the long run (graphically, the long run cost function envelopes all short-run cost functions).

Question 5, c)


73
£/output unit
ACs(y;x2''')
ACs(y;x2')
ACs(y;x2'')
AC(y)
The long-run ATC
curve is the lower envelope
of the short-run ATC curves
y

Question 5, c)

Thus, we need to know the amount of ovens that minimises costs for each level of output y.

$$MC(y,K) = ATC(y,K) \tag{89}$$

$$MC(y,K) = \frac{\partial c(y,K)}{\partial y} = \frac{\partial \frac{17y^2}{32K}}{\partial y} + \frac{0.25K}{\partial y} \tag{90}$$

$$MC(y,K) = \frac{2 \cdot 17y^{2-1}}{32K} \Longrightarrow \frac{34y}{32K} \tag{91}$$

$$MC(y,K) = \frac{34y}{32K} \tag{92}$$

Question 5, c)

$$ATC(y,K) = \frac{c(y,K)}{y} = \frac{\frac{17y^2}{32K}}{y} + \frac{0.25K}{y} \tag{93}$$

$$ATC(y,K) = \frac{c(y,K)}{y} = \frac{\frac{17\cancelto{y}{y^2}}{32K}}{\cancelto{1}{y}} + \frac{0.25K}{y} \tag{94}$$

$$ATC(y,K) = \frac{17y}{32K} + \frac{0.25K}{y} \tag{95}$$

Now we can compute the value of K^*, for all the values of y

$$ATC(y,k) = MC(y,K) \tag{96}$$

Question 5, c)

Now we can compute the value of K^*, for all the values of y

$$ATC(y,K) = MC(y,K) \tag{97}$$

$$\frac{17y}{32K} + \frac{0.25K}{y} = \frac{34y}{32K} \tag{98}$$

$$\frac{0.25K}{y} = \frac{34y}{32K} - \frac{17y}{32K} \tag{99}$$

$$\frac{K}{4y} = \frac{17y}{32K} \tag{100}$$

Question 5, c)

We continue solving this equation until we find K^* as a function of y:

$$K^2 = \frac{4 \cdot 17y^2}{32} / \sqrt{} \tag{101}$$

$$K = \sqrt{\frac{\cancel{4}^{1} \cdot 17y^2}{\cancel{32}^{8}}} \tag{102}$$

$$K^* = y\sqrt{\frac{17}{8}} \tag{103}$$

Question 5, c)

The long-run cost function of the firm is just the short-run cost function evaluated at the optimal choice of fixed factors. Thus, we need to know the amount of ovens that minimises costs for each input y.

$$C(y,K) = \frac{17y^2}{32K} + 0.25K \tag{104}$$

$$c(y) = c_s(y, k(y)) = \frac{17y^2}{32 \cdot y\sqrt{\frac{17}{8}}} + 0.25 \cdot y\sqrt{\frac{17}{8}} \tag{105}$$

$$c(y) = \frac{17\cancelto{y}{y^2}}{32 \cdot \cancelto{1}{y}\sqrt{\frac{17}{8}}} + 0.25 \cdot y\sqrt{\frac{17}{8}} \tag{106}$$

Question 5, c)

$$c(y) = \frac{\sqrt{17}\sqrt{8}y}{32} + \frac{y\sqrt{17}}{4\sqrt{8}} \tag{107}$$

$$c(y) = \frac{4\sqrt{8}\cdot\sqrt{17}\sqrt{8}y + 32\cdot y\sqrt{17}}{32\cdot 4\sqrt{8}} \tag{108}$$

$$c(y) = \frac{32\sqrt{17}y + 32\cdot y\sqrt{17}}{32\cdot 4\sqrt{8}} \tag{109}$$

$$c(y) = \frac{\cancelto{1}{64}\sqrt{17}y}{\cancelto{1}{32}\cdot\cancelto{2}{4}\sqrt{8}} \tag{110}$$

$$c(y) = \frac{1}{2}\frac{\sqrt{17}y}{\sqrt{8}} \tag{111}$$

Question 5, c)

Does it exhibit increasing, constant, or decreasing returns to scale?

Question 5, c)

Does it exhibit increasing, constant, or decreasing returns to scale?

$$c(y) = \frac{1}{2}\frac{\sqrt{17}y}{\sqrt{8}} \tag{112}$$

We divide the cost function by y in order to get our long-run ATC:

$$ATC_L = \frac{1}{2}\frac{\sqrt{17}}{\sqrt{8}} \tag{113}$$

Question 5, c)

One thing that we can deduce from the long-run ATC(y) function is that is constant. It does not depend on y:

$$ATC_L = \frac{1}{2}\frac{\sqrt{17}}{\sqrt{8}} \tag{114}$$

Therefore, this firm exhibits constant returns to scale.

Microeconomics (5SSPP217)

Seminar Problem Set 6

Felipe Torres
felipe.torres@kcl.ac.uk

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Contact & Office Hours

- My email: **felipe.torres@kcl.ac.uk**
- Office hours: Fridays from 11:00 to 12:00 hrs. Bush House (NE). 9.01

1. A perfectly competitive firm has a short-run total cost function that is $C(y) = 5 - 0.5y + 0.001y^2$ where y is output rate (units per time period).

a. Determine the firm's short run supply function:

How to answer this question?

We need to know what the Firm's short run supply function looks like

$$\Downarrow$$

How do we get the supply function using the functions that are provided in the question

We need to know what the Firm's short run supply function looks like

Answer:

It's a function of y

How to answer this question?

How do we get the supply function using the functions that are provided in the question:

Answer:

We know from slide 22 (lecture 1 that the Firm's short run supply is $MC(y_s)$, and from slide 23 for all values where $\frac{\partial MC(y^*)}{\partial y} > 0$. Thus the MC of a competitive firm is the precisely its supply function, provided that:

$$py - c_s(y) \geq 0 \text{ which is equivalent to } p \geq AVC_s(y) \qquad (1)$$

Let's obtain the marginal cost:

$$C(y) = 5 - 0.5y + 0.001y^2 \tag{2}$$

$$MC = \frac{\partial C(y)}{\partial y} = \frac{\partial 5}{\partial y} - \frac{\partial 0.5y}{\partial y} + \frac{\partial 0.001y^2}{\partial y} \tag{3}$$

$$MC = \frac{\partial C(y)}{\partial y} = -0.5 + 2 \cdot 0.001y^{2-1} \Longrightarrow -0.5 + 0.002y \tag{4}$$

$$MC = -0.5 + 0.002y \tag{5}$$

On the other hand, we know that we have to meet the following condition, where is MC is upward slopping, and the firm is making non-negative profits:

$$MC = p \geq AVC_s(y) \tag{6}$$

We also know that:

$$AVC = \frac{c_v(y)}{y} = \frac{-0.5y + 0.001y^2}{y} \implies -0.5 + 0.001y \tag{7}$$

$$MC = p \geq \frac{5}{y} - 0.5 + 0.001y \tag{8}$$

One thing that we can identify here is that our MC is always greater than our Average Variable Cost function

$$MC = -0.5 + 0.002y \tag{9}$$

Average Variable Cost:

$$AVC = -0.5 + 0.001y \tag{10}$$

$$MC > AVC \tag{11}$$

Then, for any value of y, our MC function is the firm's supply function.

b. If the firm sells its product at a price of £0.10 per unit, determine the output rate that maximises profit or minimises losses in the short term.

How to answer this question?

We need to know what output rate means

$\Downarrow$

Are they asking for y? given what?

$\Downarrow$

We need to know the decision rule whereby the firms determine how much output level will they produce for a given price

We need to know what output rate means

Answer: They are asking us for total output of an individual firm

$\Downarrow$

Are they asking for y? given what?

Answer: They are asking us for total output given a certain price (p=0.10)

$\Downarrow$

We need to know the decision rule whereby the firms determine how much output will they produce for all prices

Answer: We know from slide 22 (lecture 11), that profit-maximising firm will produce a given level of output as long as $MC = p$

Answer: We know from slide 22 (lecture 1), that profit-maximising firm will produce a given level of output as long as $MC = p$

$$MC = p \tag{12}$$

$$-0.5 + 0.002y = 0.10 \tag{13}$$

$$0.002y = 0.10 + 0.5 \Longrightarrow 0.002y = 0.6 \tag{14}$$

$$0.002y = 0.6 \Longrightarrow y = \frac{0.6}{0.002} \tag{15}$$

$$y = 300 \tag{16}$$

Answer: When the firms produces y $= 300$, the $MC > AVC$, thus we know is better than shutting down.

Question 1, c

c) If input prices increase and cause the cost functions to become $C(y) = 5 - 0.10y + 0.002y^2$, what will the new equilibrium output and profit be? Explain intuitively what happened to output and profits when input prices increased

How to answer this question?

We need to know what equilibrium means and how profits are computed

$\Downarrow$

We need to know the equations that we need to use to find equilibrium and the profit function

$\Downarrow$

We need to find equilibrium and compute the profits in equilibrium

$\Downarrow$

4) We need compare output levels/profits with the old and new cost functions, and understand the intuition behind this change.

How to answer this question?

We need to know what equilibrium means and how profits are computed

Answer: We find equilibrium when the marginal revenue (or price) is equal to the marginal cost (MC)

We need to know the equations to find equilibrium and the profit function

Answer: We know that $p = MC$ and $\pi = py - c_s(y)$

We need to find equilibrium and compute the profit in equilibrium

Answer: We know that $p = MC$ and $MC = \frac{\partial c_s(y)}{\partial y}$

4) We need compare output levels/profits with the old and new cost functions, and understand the intuition behind this change

Answer: We need to understand that happens with output when it becomes costlier to produce a given level of output.

First, we see that our variable costs doubled:

$$C(y) = 5 - 0.10y + 0.002y^2 \tag{17}$$

Let's find our new MC:

$$\frac{\partial c_s(y)}{\partial y} \longrightarrow MC = \frac{\partial 5}{\partial y} - \frac{\partial 0.1y}{\partial y} + \frac{\partial 0.002y^2}{\partial y} \tag{18}$$

$$MC = 0 - 0.12 \cdot 0.02y^{2-1} \Longrightarrow MC = -0.1 + 0.004y \tag{19}$$

$$MC = -0.1 + 0.004y \tag{20}$$

Our new $AVC = -0.1 + 0.002y$, so our supply function is the firm's MC for all values of y.

In equilibrium, with p=0.1:

$$p = -0.1 + 0.004y \tag{21}$$

$$0.1 = -0.1 + 0.004y \tag{22}$$

$$0.1 = -0.1 + 0.004y \implies 0.2 = 0.004y \implies y = \frac{0.2}{0.004} \tag{23}$$

$$y^* = 50 \tag{24}$$

Again the MC is above the AVC.

Because the inputs are now costlier, output and profits will go down but that still would be better than shutting down.

Question 2,a

Laura owns a firm of internet services with the following short-run cost curve: $C(y,K) = \frac{25y^3}{K^{\frac{2}{3}}} + rK$ where y is Laura's output level, K, is the number of servers she leases and r is the lease price of servers. Laura's short-run marginal cost function is: $MC(y,K) = \frac{50y}{K^{\frac{2}{3}}}$. Currently, Laura leases k=8 servers, and Laura can sell all her internet services for £500 per unit.

a. Find Laura's short-run profit maximising level of output if r= £15. Calculate Laura's profits

Did anyone notice something weird in this question?

The correct marginal cost is equal to:

$$MC = \frac{25}{K^{\frac{2}{3}}} \frac{\partial y^3}{\partial y} + \frac{\partial rk}{\partial y} \tag{25}$$

$$MC = \frac{3 \cdot 25}{K^{\frac{2}{3}}} y^{3-1} + \cancelto{0}{\frac{\partial rk}{\partial y}} \tag{26}$$

$$MC = \frac{75y^2}{K^{2/3}} \tag{27}$$

We will use this marginal cost to solve this problem.

How to answer this question?

First, we have the short cost function, and we need to know how we can use this function to find Laura's profit

$$\Downarrow$$

We need to compute/use the MC to calculate the output level of Laura's firm for a given price

$$\Downarrow$$

We need to find the profit function and calculate for a given level of output and price

How to answer this question?

First, we have the short cost function, we need to know how we can use this function to find Laura's profit

$$\Downarrow$$

We need to compute/use the MC to calculate the output for a given price

Answer: To determine the optimal output level, we need to first equate marginal cost to the market price, with k = 8, that is:

$$MC(y,8) = \frac{75y^2}{K^{\frac{2}{3}}} \longrightarrow \frac{75y^2}{8^{\frac{2}{3}}} \tag{28}$$

$$MC(y,8) = \frac{75y^2}{4} \tag{29}$$

They tell us that Laura can sell her internet services at £500 per unit, thus p = 500:

$$MC(y,8) = p \Longrightarrow \frac{75y^2}{4} = 500 \tag{30}$$

$$75y^2 = 500 \cdot 4 \Longrightarrow 75y^2 = 2000 \Longrightarrow y^2 = \frac{2000}{75} \tag{31}$$

$$y = 5.16 \text{ or } 4\sqrt{\frac{5}{3}} \tag{32}$$

Let's check that the MC is above the AVC

$$AVC = \frac{C(y,K)}{y} = \frac{\frac{25y^3}{4}}{y} \Longrightarrow \frac{\frac{25\cancelto{y^2}{y^3}}{4}}{\cancelto{1}{y}} \tag{33}$$

$$AVC = \frac{25y^2}{8^{\frac{2}{3}}} \Longrightarrow \frac{25y^2}{4} \text{ vs } MC = \frac{75y^2}{4} \tag{34}$$

We know that a firm maximises its profits as long as $P \geq AVC$. Let's check that this is met at y = 5.16 (or $4\sqrt{\frac{5}{3}}$)

$$AVC(5.16) = \frac{25 \cdot 5.16^2}{4} = \frac{665.64}{4} \Longrightarrow 166.67 \tag{35}$$

Since $p > 166.67$, $500 > 166.67$, Laura indeed maximises profits at y = 5.16.

Now that we know y, p, and the cost function, we can calculate Laura's profits:

What is Laura's profit function?

$$\pi = py - c_s \Longrightarrow 500 \cdot 4\sqrt{\frac{5}{3}} - 25\frac{(4\sqrt{\frac{5}{3}})^3}{4} - 15 \cdot 8 = 1601.3 \qquad (36)$$

Then, Laura's profit is £1601.3

Question 2,b

b. If the lease rate of internet server rises to r' = £20, how does Laura's short run optimal output and profit change?

b. If the lease rate of internet server rises to r' $=$ £20, how does Laura's short run optimal output and profit change?

How to approach this question?

We need to think how this price change affects Laura's AVC and MC, and profits.

Her AVC and MC are unaffected by the lease rate. We can see this in the AVC and MC functions:

$$AVC = \frac{25y^2}{4} \tag{37}$$

$$MC(y, 8) = \frac{75y^2}{4} \tag{38}$$

What about her profits?

What about her profits?

$$\pi = py - c_s(y, K) \Longrightarrow py - c_s(y, K) \tag{39}$$

$$\pi = 500 \cdot 4\sqrt{\frac{5}{3}} - 25\frac{(4\sqrt{\frac{5}{3}})^3}{4} - 20 \cdot 8 = 1561.3 \tag{40}$$

Thus, the £5 increase in the rental rate reduced Laura's short run profits by £40.

Question 3, a

a. The demand for pizzas in the local market is given by: $Y_D = 25000 - 1500P$. There are 100 identical pizza firms currently in the market. The long-run cost function for each pizza firm is: $C(y, w) = \frac{10}{7}wy$, where w is the wage rate pizza firms pay for a labour hour and y is the number of pizzas produced.

a. Find the long-run supply function for each firm

a. Find the long-run supply function for each firm. How to answer this question?

We know how firms set up their long-term supply function

We know how firms set up their long-term supply function

Answer We know that Firm's Long-Run Supply Decision is:

$$\max \pi(y) = py - c(y) \tag{41}$$

$$MC(y) \text{ is the long-run supply decision function.} \tag{42}$$

$$\frac{\partial MC}{\partial y} > 0 \text{ and } p \geq \frac{c(y)}{y} = AC(y) \tag{43}$$

Additionally, the firm's economic profit level must not be negative:

$$p \geq \frac{c(y)}{y} = AC(y) >= 0 \tag{44}$$

$$AC(y,w) = \frac{c(y,w)}{y} = \frac{\frac{10wy}{7}}{y} \tag{45}$$

$$AC(y,w) = \frac{c(y,w)}{y} = \frac{10wy}{7} \cdot \frac{1}{y} \tag{46}$$

$$AC(y,w) = \frac{c(y,w)}{y} = \frac{10w\cancelto{1}{y}}{7} \cdot \frac{1}{\cancelto{1}{y}} \tag{47}$$

$$AC(y,w) = \frac{10w}{7} \tag{48}$$

Let's now get the MC:

$$MC = \frac{\partial C(y,w)}{\partial y} \longrightarrow MC = \frac{10}{7}\frac{\partial wy}{\partial y} \tag{49}$$

$$MC = \frac{10w}{7} \tag{50}$$

$$\frac{\partial MC}{\partial y} = 1 \text{ which is positive}^1 \tag{51}$$

Thus, the long-run supply function is $\frac{10w}{7}$. $(AC = \frac{10w}{7})$

[1] This means that the MC is upward sloping

Question 3, b

b. If the current wage rate is w=£7 and the industry is competitive, calculate the optimal output of each firm given that each firm produces the same level of output. Do you anticipate firms entering or exiting the pizza industry?

How to tackle this question?

We need to know how to compute the optimal output of each firm

$$\Downarrow$$

We need to compute/use the MC and calculate the output for a given price

$$\Downarrow$$

We need find the profit function and calculate profits for a given level of output

They tell us that $w = 7$

$$MC = \frac{10w}{7} \Longrightarrow \frac{10 \cdot 7}{7} \tag{52}$$

$$MC = \frac{10 \cdot \cancelto{1}{7}}{\cancelto{1}{7}} \tag{53}$$

$$MC = 10 \tag{54}$$

We know that the price that the firm is willing to sell is the MC.

If the price is equal to 10. We can calculate the quantity demanded:

$$Y^D = 25,000 - 1,500P \tag{55}$$

$$Y^D = 25,000 - 1,500 \cdot 10 \tag{56}$$

$$Y^D = 10,000 \tag{57}$$

$$Y^D = 10,000 \tag{58}$$

Since there are 100 firms in the industry and they divide the industry output equally, each firm is producing 100 pizzas each period.

Question 3,b - second part

Do you anticipate firms entering or exiting the pizza industry?

Do you anticipate firms entering or exiting the pizza industry?

- To determine whether there are firms entering or exiting, we need to compute whether firms are yielding positive profits:
- The average cost per pizza in the long run is equivalent to the price firms receive, thus, firms are earning only the normal profit. We can proof this mathematically:

Proof:

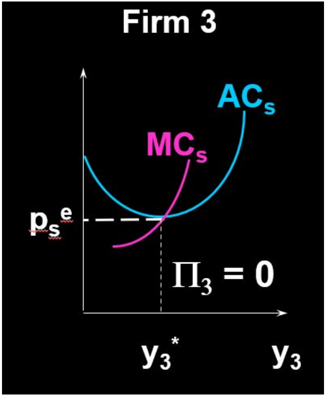
$$AC = \frac{C(y,w)}{y} \longrightarrow AC = \frac{10wy}{7} \tag{59}$$

$$AC = \frac{10 \cdot \cancelto{1}{7}}{\cancelto{1}{7}\, y} \Longrightarrow 10 \tag{60}$$

$$AC = 10 = MC \tag{61}$$

Thus, the firms are not making profits.

We are in this scenario:



This implies there is no incentive for firms to enter or exit the industry

Question 3, c

Suppose that the wage rate increases to £8.40. Calculate the optimal output for each of the 100 firms. Do you anticipate firms entering or exiting the pizza industry? What happens to the market output of pizzas with the higher wage rate? What happens to the market price for pizza?

Calculate the optimal output for each of the 100 firms.

We have a MC, we need to calculate the MC with $w = 8.4$ in order to obtain the optimal output for each of the 100 firms.

$$MC = \frac{10w}{7} \longrightarrow MC = \frac{10 \cdot 8.4}{7} \tag{62}$$

$$MC = \frac{10 \cdot 8.4}{7} \tag{63}$$

$$MC = 12 \tag{64}$$

This implies that in the long-run, the market price of pizza will be £12. At this price, consumers demand for pizzas is 7,000.

$$Y^D = 25,000 - 1,500P \tag{65}$$

$$Y^D = 25,000 - 1,500 \cdot 12 \tag{66}$$

$$Y^D = 7,000 \tag{67}$$

Question 3, c - second part

Do you anticipate firms entering or exiting the pizza industry?

The optimal output for the 100 firms is 70 pizzas per firm. Since this is also a long-run equilibrium, there is no incentive for firms to enter or exit the industry.

What happens to the market output of pizzas with the higher wage rate? What happens to the market price for pizza?

At the higher wage rate, the market output of pizzas decline. The market price for pizzas increases by 20% (from £10 to £12 when the wage rate increases by 20% (from £7 to £8.4).

Question 4

4. On an island there are 100 potential boat builders, numbered $i = 1,...,100$. Each can build up to 12 boats a year, but anyone who goes into the boat-building business must pay a fixed cost of £11. Marginal costs differ from builder to builder. Where y denotes the number of boats built per year, boat builder i has a total cost function $c(y) = 11 + iy$. If the market price for boats is £30 for each boat built, how many boats will be built?

How to tackle this question?

First, we need to find how many firms will enter into the boat building industry.

$$\Downarrow$$

We need to know under which rule firms decide whether to enter or exit the industry

$$\Downarrow$$

We need to know how to compute the total output given the number of firms in the industry

Answer: The first step is to find how many boats will be build. We need to figure out how many firms will enter the industry. We know that for the short-run:

$$\Pi(y) = py - F - c_v(y) \geq 0 \Longrightarrow p \geq AVC_s(y) \tag{68}$$

$$c(y) = 11 + iy \text{ then } MC = \frac{\partial c(y)}{\partial y} = \frac{\partial 11 + iy}{\partial y} \tag{69}$$

$$MC = \frac{\partial 11 + iy}{\partial y} \Longrightarrow \cancelto{0}{\frac{\partial 11}{\partial y}} + i \tag{70}$$

$$MC = i \tag{71}$$

This means that firm i (i = 1,..100):

- $MC_1 = 1$
- $MC_2 = 2$
- $MC_3 = 3$
- ....
- $MC_i = i$

Firms will produce as long as $p \geq MC$. They tell us that the market price is £30.

- $MC_1 = 1$, enters;
- $MC_2 = 2$, enters;
- $MC_3 = 3$, enters;
- ....
- $MC_29 = 29$, enters;
- $MC_30 = 30$, indifferent between shutting down or producing
- $MC_31 = 31$, will not produce; as it will make a loss.

Then, either 29 or 30 boat builders will build boats.

How many boats will be built?

How do we calculate the total output?

They tell us in the question that each boat builder can build up to 12 boats a year.

If 29 boat builders produce in this industry:

$$\text{Total output} = 29 \cdot 12 = 348 \tag{72}$$

If 30 boat builders produce in this industry:

$$\text{Total output} = 30 \cdot 12 = 360 \tag{73}$$

Question 5, a

The market for wheat consists of 500 identical firms, each with short-run total cost function $C(y) = 90,000 + 0.00001y^2$ where y is measured in bushels per year. The market demand curve for wheat is $Y_D = 90,000,000 - 20,000,000P$, where Y_D is again measured in bushels and P is the price per bushel.

a) Determine the short-run equilibrium price and quantity that would exist in the market

How to tackle this question?

First, we need to know how to obtain the market supply curve

$$\Downarrow$$

Then, we need to know how to find the equilibrium price and quantity, using both the market supply function and market demand function

First, we need to know how to obtain the market supply curve:

We know from slide 5, lecture 12, that the industry/market short-run supply function is:

$$S(p) = \sum_{i=1}^{n} S_i(p) \tag{74}$$

Thus, we need to aggregate the supply function for the 500 firms. This also means that we need to find the supply of each individual firm.

Question: Is the supply function either?

- Quantity produced = Price , for example; Q = P, or
- Price = Quantity produced, for example P = Q.

It's the quantity supplied as a function of price.

In order to find the individual supply function, we first need to get the individual marginal cost where we will get y as function p:

$$C(y) = 90,000 + 0.00001y^2 \tag{75}$$

$$MC = \frac{\partial C(y)}{\partial y} = \frac{\partial 90,000}{\partial y} + \frac{\partial 0.00001y^2}{\partial y} \tag{76}$$

$$MC = \cancelto{0}{\frac{\partial 90,000}{\partial y}} + 2 \cdot 0.00001y^{2-1} \Longrightarrow MC = 0 + 0.00002y \tag{77}$$

$$MC = 0.00002y^2 \tag{78}$$

---

[2] The second derivative of the MC is equal to 0.0002, thus is upward-slopping.

$$MC = 0.00002y = P \tag{79}$$

$$0.00002y = P \longrightarrow y = 50,000P \tag{80}$$

This is the individual supply function:

$$y = 50,000P \tag{81}$$

Final step, we aggregate across the 500 firms

$$y_s = 50,000P \tag{82}$$

$$Y_s = 500(50,000)P = 25,000,000P \tag{83}$$

$$Y_s = 25,000,000P \tag{84}$$

What is the next step to find short-run equilibrium price and quantity?

We need to equate Y_s and Y_D to determine the market price

$$Y_S = Y_D \tag{85}$$

$$25,000,000P = 90,000,000 - 20,000,000P \tag{86}$$

which yields $P^* = £2.00$ and $Y^* = 50,000,000$.

Question 5, b

b. Calculate the profit maximising quantity for the individual firm. Calculate the firm's short-run profit (loss) at that quantity

We know that we can get the profit maximising quantity for the individual firm using the MC.

$$MC = 0.00002y = P \tag{87}$$

$$MC = 0.00002y = 2 \Longrightarrow y = \frac{2}{0.00002} \tag{88}$$

$$y = 50,000,000 \tag{89}$$

We know that we can get the profit maximising quantity for the individual firm using the MC.

$$MC = 0.00002y = P \tag{90}$$

$$MC = 0.00002y = 2 \Longrightarrow y = \frac{2}{0.00002} \tag{91}$$

$$y = 100,000 \tag{92}$$

Calculate the firm's short-run profit (loss) at that quantity:

$$\pi = TR - TC = py - c(y) \tag{93}$$

$$\pi = TR - TC = 2 \cdot 100,0000 - (90,000 + 0.000001 \cdot 100,000^2) \tag{94}$$

$$\pi = TR - TC = 200,000 - 190,000 = 10,000 \tag{95}$$

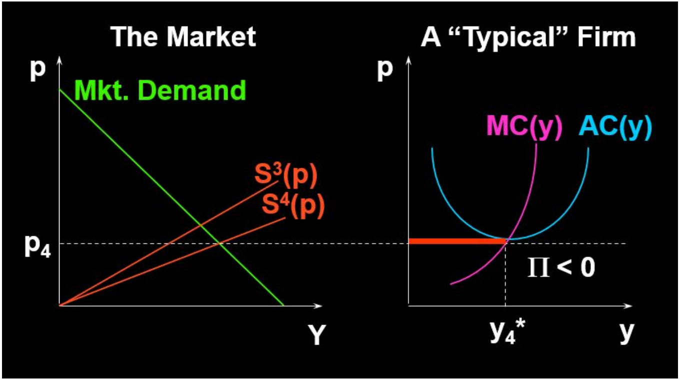
$$\pi = 10,000 \tag{96}$$

Question 5, c

c. Assume there are no barriers to entry or exit in the market. Describe the expected long-run response to the conditions described in part b. (The cost function for the firm may be regarded as an economic cost function that captures all implicit and explicit costs.)

Answer: Firms are earning economic profit so we would expect entry to occur, causing the market supply curve to shift rightward. As the market supply curve shifts rightward, price falls, which in turn causes each firm to reduce its output. This will continue until we reach long-run equilibrium at zero profit.

Answer: We saw can see this in slide 30, lecture 12, here a modified version in the following figure:



Question 6, a

The long run cost function for a typical firm in a competitive industry is given by:

$$c(y) = \begin{cases} 0 \text{ for } y = 0 \\ 100 + y^2 \text{ for } y > 0 \end{cases} \tag{97}$$

Market demand for the industry product is given by $Y_D = 500 - 10P$

a. Assuming all firms have identical costs, find the long run competitive equilibrium price, total quantity, long run number of firms, and the quantity produced by each firm in the long run

How to tackle this question?

First, we need to know what is the individual supply function for each firm

$\Downarrow$

We need to know what what is the level of output a competitive firm will choose to produce

$\Downarrow$

We need to know how to find the competitive prices using the MC, and how to use the market demand function to identify the long run quantity demanded.

How to tackle this question?

First, we need to know what is the individual supply function for each firm

We know that each firm decides to supply in the long-run (see slide 44, lecture 1):

$$Max \Pi(y) = py - c(y) \tag{98}$$

$$y \geq 0$$

$$p = MC(y) \text{ and } \frac{\partial MC}{\partial y} > 0 \tag{99}$$

Additionally, the firm's economic profit level must be non-negative

$$MC(y) = p \geq AC(y) \tag{100}$$

Let's get first the supply of the individual firm, which is the MC above the AVC for $y > 0$:

$$c(y) = 100 + y^2 \implies \frac{\partial c(y)}{\partial y} = \frac{\partial 100}{\partial y} + \frac{\partial y^2}{\partial y} \tag{101}$$

$$MC = \frac{\partial 100}{\partial} + \frac{\partial y^2}{\partial y} \implies MC = \cancelto{0}{\frac{\partial 100}{\partial}} + 2 \cdot y^{2-1} \tag{102}$$

$$MC = 2y \tag{103}$$

Let's meet the second condition $p \geq AC(y)$ for $y > 0$

$$c(y) = 100 + y^2 \Longrightarrow \frac{c(y)}{y} = AC \tag{104}$$

$$AC = \frac{100}{y} + \frac{y^2}{y} \Longrightarrow AC = \frac{100}{y} + y \tag{105}$$

$$AC = \frac{100}{y} + y \tag{106}$$

You can see that the AC function is a U-shaped function. For low values of y, AC is high, whereas, for high values of y, $\frac{100}{y}$ goes to zero, but then increases as y increases.

For low values of y, for example y = 1

$$AC = \frac{100}{\cancel{y}} \quad + \cancel{y} = 101 \tag{107}$$

For values y = 10

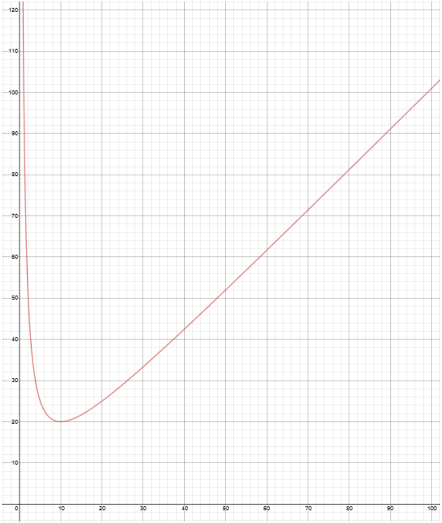
$$AC = \frac{100}{\cancel{10}} \quad + 10 = 20 \tag{108}$$

For values y = 100

$$AC = \frac{\cancel{100}^{\nearrow 1}}{\cancel{100}} + 100 = 101 \tag{109}$$

For a given value $y = \infty$

$$AC = \frac{\cancel{100}^{\nearrow 0}}{\cancel{y}} + \cancel{y}^{\nearrow \infty} = \infty \tag{110}$$


120
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90
100

Now that we know that the AC function is U-shaped, the individual supply function will be:

$$MC \geq AC \implies 2y = \frac{100}{y} + y \tag{111}$$

$$2y = \frac{100}{y} + y \implies 2y - y = \frac{100}{y} \tag{112}$$

$$y = \frac{100}{y} \implies y \cdot y = 100 \implies y^2 = 100 \tag{113}$$

$$y^2 = 100 \implies \mathbf{y = 10} \tag{114}$$

This corresponds to the quantity with the minimum Average Cost.

So, at the minimum, the MC cost is equal to:

$$MC = 2 \cdot 10 \Longrightarrow MC = 20 \tag{115}$$

The AC at the minimum is also 20

$$MC = AC = 20 \tag{116}$$

We also know that $P = MC$, thus the long rung competitive price will be $\mathbf{P = 20}$

Now that we know that the price (P=20), we can identify the quantity demanded:

$$Y_D = 500 - 10P \implies Y_D = 500 - 10 \cdot 20 \tag{117}$$

$$Y_D = 500 - 10 \cdot 20 \implies Y_D = 500 - 200 \implies Y_D = 300 \tag{118}$$

$$Y_D = 300 \tag{119}$$

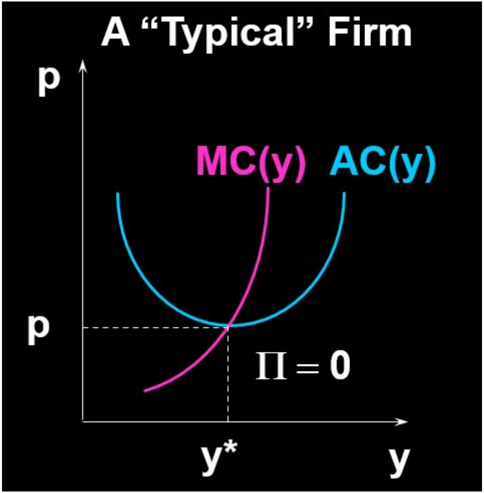
This is the quantity demanded Y = 300. Thus, each firm will produce $y = \frac{300}{n}$

Assuming all firms produce exactly the same amount, the profit for an individual firm will then be the profit function divided by n firms:

$$\pi_n = \frac{Py}{n} - F - \frac{c_v(y)^2}{n} \Longrightarrow \pi_n = \frac{20 \cdot 300}{n} - 100 - (\frac{300}{n})^2 \quad (120)$$

How much profits firms get when MC = AV? (See slides 26-31, lecture 1, also the fixed cost is not divided across firms, as each of them has to pay for it.

When MC = AC, profits are zero:



Then, $\pi_n = 0$

$$\pi_n = \frac{Py}{n} - 100 - (\frac{300}{n})^2 \implies 0 = \frac{6000}{n} - 100 - \frac{9000}{n^2} \tag{121}$$

$$0 = \frac{6000}{n} - 100 - \frac{9000}{n^2} / \text{ We multiply both sides by } n^2 \tag{122}$$

$$0 = 6000n - 100n^2 - 9000 \tag{123}$$

$$0 = 6000n - 100n^2 - 9000 \tag{124}$$

Using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{125}$$

Let's arrange this equation first:

$$0 = -100n^2 + 6000n - 9000 \tag{126}$$

- a = -100
- b = 6000
- c = -9000

Using the quadratic formula:

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \implies n = \frac{-6000 \pm \sqrt{6000^2 - 4 \cdot -100 \cdot -9000}}{2 \cdot -100} \tag{127}$$

n = 30, then, 30 firms will enter when $\pi = 0$; hence, each firm will produce 10 units.

Question 6, b

b) Suppose that the government restricts entry into this market. In particular, assume that the government requires that each firm obtains a license to participate in the market, and that only 20 licenses are issued. Assuming all firms have identical cost structures, find the long run competitive equilibrium price, total quantity, quantity produced by each firm, and profits of each firm. How much would a firm be willing to pay for a license?

Because there is no free entry, the market price will not converge to the minimum AC (MC = AC). But, firms will produce MC = P

$$MC = P \Longrightarrow 2y = P \tag{128}$$

Hence, each firm will produce:

$$2y = P \Longrightarrow y = \frac{P}{2} \tag{129}$$

As there will be 20 firms in the market, total supply will be?

As there will be 20 firms in the market, total supply will be?

$$Y^S = y \cdot 20 \Longrightarrow \frac{P}{2} \cdot 20 \tag{130}$$

$$Y^S = 10P \tag{131}$$

When this supply meets the demand, we can find the price in equilibrium:

$$Y^S = Y^D \Longrightarrow 10P = 500 - 10P \tag{132}$$

$$20P = 500 \Longrightarrow P = 25 \tag{133}$$

Given that we have price in equilibrium P = 25, this corresponds to a total quantity of:

$$Y^S = 10P \tag{134}$$

$$Y^S = 10 \cdot 25 \tag{135}$$

$$Y^S = 250 \tag{136}$$

Given that there will be 20 firms, each firm will produce $\frac{250}{20} = 12.5$

Now that we know prices and quantities, we can calculate profits for each firm:

$$\pi = py - c(y) \Longrightarrow 25 \cdot 12.5 - 100 + (12.5)^2 \tag{137}$$

$$\pi = 25 \cdot 12.5 - 100 + (12.5)^2 \Longrightarrow \pi = 312.5 - 100 - 156.25 \tag{138}$$

$$\pi = 56.25 \tag{139}$$

We know that the price of the license will be completed up to the point where it reaches £56.25 and the economic profits of an incumbent firm are zero; or, in other words, up to the point where the owner of a license is indifferent between selling the license and operating in the market at the current market price.

Office hours: Every Friday from 11:00 to 12:00

Bush House, NE. 9.01

felipe.torres@kcl.ac.uk

Microeconomics (5SSPP217)

Seminar Problem Set 7

Felipe Torres
felipe.torres@kcl.ac.uk

June 4, 2025

Contact & Office Hours

- My email: **felipe.torres@kcl.ac.uk**
- Office hours: Fridays from 11:00 to 12:00 hrs, Bush House (NE) 9.01

Question 1a

1. Monopolist faces the demand curve P = 11 - Q, where P is measured in £ per unit and Q in thousands of units. The monopolist has a constant average cost of £6.

a. Draw the average and marginal revenue curves and the average and marginal cost curves. What are the monopolist's profit-maximising price and quantity? What is the resulting profit? Calculate the firm's degree of market power using the Lerner index

We know that we can derive from the demand function $P(Q) = a - bQ$, the Marginal Revenue function

$$MR(Q) = p(Q) + Q \cdot p'(Q) \tag{1}$$

$$MR(Q) = a - b \cdot Q + Q \cdot (-b) \tag{2}$$

$$MR(Q) = a - 2b \cdot Q \tag{3}$$

Our demand function is $P(Q) = 11 - Q$:

$$a = 11 \tag{4}$$

$$b = 1 \tag{5}$$

Thus, our MR(Q) is:

$$MR(Q) = 11 - 2 \cdot 1 \cdot Q \Longrightarrow MR(Q) = 11 - 2Q \tag{6}$$

$$MR(Q) = 11 - 2Q \tag{7}$$

Average revenue is equal to $AR = \frac{TR}{Q}$:

$$AR = \frac{TR}{Q} \tag{8}$$

$$AR = \frac{P(Q)Q}{Q} \implies \frac{P\cancelto{1}{Q}}{\cancelto{1}{Q}} \tag{9}$$

Thus, our AR is equal to:

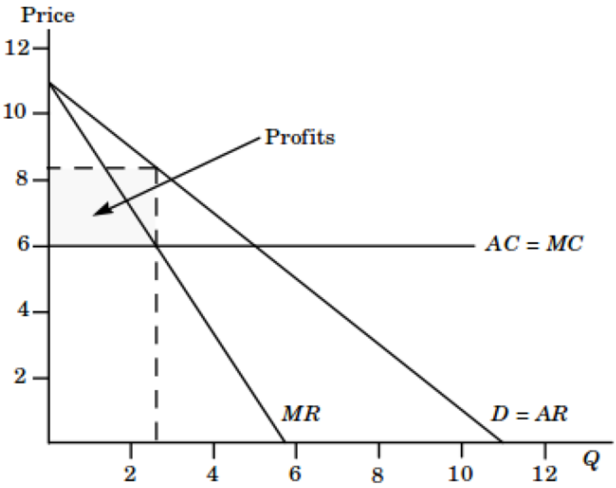
$$AR = P(Q) \text{ and } \implies P(Q) = 11 - Q \tag{10}$$

They tell us that the constant average cost is £6. Then:

$$MC = 6 \tag{11}$$

$$AC = 6 \tag{12}$$

Then, we have MC = AV = 6, and a downward sloping MR function 11- 2Q.



1.a What are the monopolist's profit-maximising price and quantity?

We know that a monopoly firm maximises profits when MR = MC (slide 15, lecture 13)

$$MR(Q) = MC(Q) \tag{13}$$

Thus, we replace our MC and MR

$$11 - 2Q = 6 \Longrightarrow Q^* = 2.5 \tag{14}$$

They tell us that Q is in thousand of units. Thus $Q^* = 2,500$

We can obtain the market price by plugging the monopolistic optimal quantity Q^* into the demand function:

$$P = 11 - Q \implies P = 11 - 2.5 \implies P^* = £8.5 \tag{15}$$

$$P^* = £8.5 \tag{16}$$

We know that the profits are calculated as follow (slide 9, lecture 13):

$$\pi(Q) = p(Q) \cdot Q - C(Q) = R(Q) - C(Q) \tag{17}$$

We know that the optimal quantity for a monopoly $Q^* = 6$ and $P^* = £2.5$.

$$\pi(Q^*) = P^*(Q^*) \cdot Q^* - C(Q^*) = £8.5 \cdot 2.5 - £6 \cdot 2.5 \tag{18}$$

$$\pi(Q^*) = £8.5 \cdot 2.5 - £6 \cdot 2.5 \tag{19}$$

$$\pi(Q^*) = £6.25 \tag{20}$$

$$\pi(Q^*) = £6.25 \tag{21}$$

Or

$$\pi(Q^*) = £6250 \tag{22}$$

Calculate the firm's degree of market power using the Lerner index

We know that the Lerner Index is:

$$\text{Lerner Index} = \frac{P - MC}{P} \tag{23}$$

$$\text{Lerner Index} = \frac{8.5 - 6}{8.5} \tag{24}$$

$$\text{Lerner Index} = 0.294 \tag{25}$$

Question 1,b

b. A government regulatory agency sets a price ceiling of £7 per unit. What quantity will be produced, and what will the firm's profit be? What happens to the degree of market power?

To determine the effect of the price ceiling (£7) on the quantity demanded, substitute the ceiling price into the demand equation:

$$P = 11 - Q \Longrightarrow 7 = 11 - Q \tag{26}$$

$$Q = 4 \tag{27}$$

Q = 4,000 rather than 2,500.

Also, the monopolist will choose to sell its product at the price (£7), that is greater than the constant marginal cost £6. Then, profits are equal to total revenue minus total csots:

- P = £7
- Q = 4
- C(Q) = 6. This doesn't change

We know that that our profit function is equal to:

$$\pi = p_{gov} \cdot Q - C(Q) \tag{28}$$

$$\pi = £7 \cdot 4000 - £6 \cdot 4000 \tag{29}$$

$$\pi = £4000 \tag{30}$$

What happens to the degree of market power?

$$\text{Lerner Index} = \frac{P - MC}{P} \tag{31}$$

$$\text{Lerner Index} = \frac{7 - 6}{7} \tag{32}$$

$$\text{Lerner Index} = 0.143 \tag{33}$$

Question 1,c

c. What price ceiling yields the largest level of output? What is that level of output? What is the firm's degree of monopoly power at this price?

c) If the regulatory authority sets a price below £6, the monopolist would prefer to go out of business because it cannot cover its average variable costs.

At any price above \$6, the monopolist would produce less than the 5000 units that would be produced in a competitive industry.

$$P = MC \implies P = 11 - Q \implies 6 = 11 - Q \implies Q = 5000 \qquad (34)$$

Therefore, the regulatory agency should set a price ceiling of \$6 ε and ε is very small, thus making the monopolist face a horizontal effective demand curve up to Q = 5 (i.e., 5000 units). To ensure a positive output (so that the monopolist is not indifferent between producing 5000.

What is the firm's degree of monopoly power at this price?

The degree of monopoly power is equal to:

$$\text{Lerner Index} = \frac{P - MC}{P} \tag{35}$$

$$\text{Lerner Index} = \frac{(6 + \varepsilon) - 6}{(6 + \varepsilon)} = 0 \tag{36}$$

$$\text{Lerner Index} = \frac{\varepsilon}{(6 + \varepsilon)} = 0 \tag{37}$$

$$\text{If } \varepsilon \longrightarrow 0, \text{ then } \frac{\varepsilon}{(6 + \varepsilon)} \longrightarrow 0 \tag{38}$$

Question 2.a

A firm has two factories for which costs are given by:

- Factory 1: $C_1(Q_1) = 10Q_1^2$
- Factory 2: $C_2(Q_2) = 20Q_1^2$

The firm faces the following demand curve: P = 700 - 5Q, where Q is total output i.e., Q = Q1 + Q2.

a. Calculate the values of Q_1, Q_2, Q, and P that maximise profit.

How to approach this question?

1. Well, we know that a firm wont' produce any products if the MR is below the MC.

2. Thus, we need to know the MR.

3. We also need to compute the MC of each factory.

We know that we can get the Marginal Revenue function from the demand function:

$$P(Q) = a - b \cdot Q \tag{39}$$

We know that for a linear demand curve our MR is: $P = 700 - 5Q$

$$MR(Q) = a - 2b \cdot Q \Longrightarrow MR(Q) = 700 - 10Q \tag{40}$$

$$MR(Q) = 700 - 10Q \tag{41}$$

Now, let's compute the MC of each function. The MC for factory 1:

$$MC_1 = \frac{dC_1}{Q_1} = 20Q_1 \tag{42}$$

$$MC_1 = \frac{\partial C(Q_1)}{\partial Q_1} \implies \frac{\partial 10Q_1^2}{\partial Q_1} \implies \frac{2 \cdot \partial 10Q_1^{2-1}}{\partial Q_1} \tag{43}$$

$$MC_1 = 20Q_1 \tag{44}$$

The MC for factory 2:

$$MC_2 = \frac{dC_2}{Q_2} = 40Q_2 \tag{45}$$

We know that the output should be divided between the two factories so that the marginal cost is the same in each factory. In addition, the marginal cost must equate the marginal revenue:

$$MC_1 = MC_2 = MR \tag{46}$$

$$20Q_1 = 40Q_2 = 700 - 10(Q_1 + Q_2) \tag{47}$$

Solving Q_1 as function of Q_2:

$$20Q_1 = 40Q_2 \Longrightarrow Q_1 = \frac{40Q_2}{20} \Longrightarrow Q_1 = 2Q_2 \tag{48}$$

Then, we can replace Q_1 in the MR function and solve for Q_2

$$40Q_2 = 700 - 10(2Q_2 + Q_2) \Longrightarrow 40Q_2 = 700 - 30Q_2 \tag{49}$$

$$70Q_2 = 700 \tag{50}$$

$$Q_2^* = 10 \text{ then } Q_1^* = 20 \tag{51}$$

Then, the optimal quantity where the monopoly maximise its profits is:

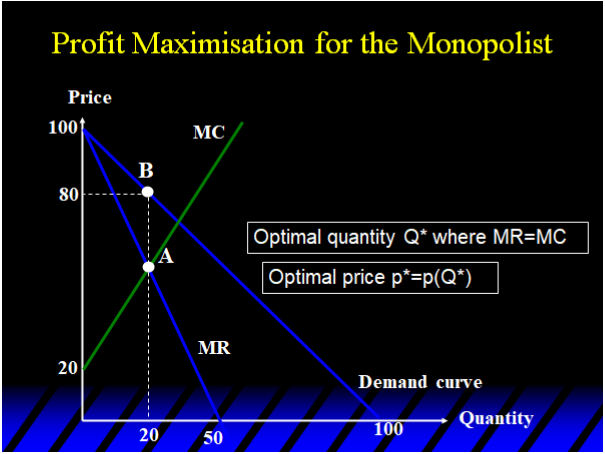
$$Q^* = Q_1 + Q_2 = 20 + 10 = 30 \tag{52}$$

$$P = 700 - 5Q \Longrightarrow P = 700 - 5 \cdot 30 \Longrightarrow P = £550 \tag{53}$$

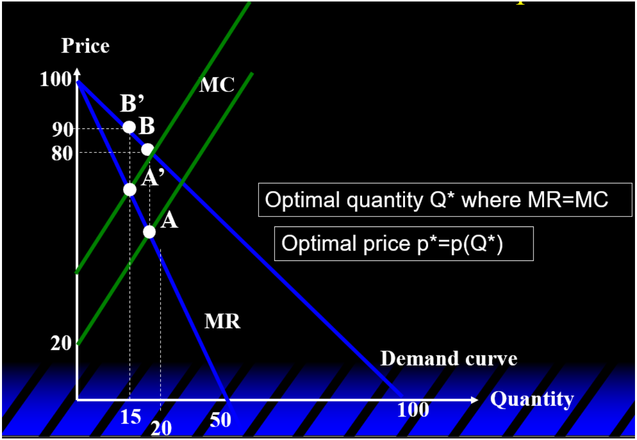
Question 2,b

b. Suppose that labour costs increase in Factory 1 but not in Factory 2. How should the firm adjust (i.e., raise, lower, or leave unchanged) the following: Output in Factory 1? Output in Factory 2? Total output? Price?

An increase in labour costs will lead to an increase in MC_1, which will meet the marginal revenue curve at lower quantity and higher marginal revenue. Let's see this in a graph



A higher marginal revenue implies necessarily a lower total quantity Q and, consequently a higher market price.



Finally, since MC_2 must equate now to a larger value, i.e the new MC_1, Q_2 will be greater than at the original level (but not enough to compensate the reduction in Q_1)

$$MC_1' = MC_2 \implies {Q_2}' > Q_2 \tag{54}$$

Total output: Let's remember that $C(Q_2)$ had a higher marginal cost, so it won't be able to compensate the drop on production from factory 1.

$$Q^{'} = {Q_1}' + {Q_2}' \tag{55}$$

$$Q^{*} > Q^{'} \tag{56}$$

Question 3.a

3. The employment of graduate teaching assistants (GTAs) by major universities in Central London can be characterised as a monopsony. Suppose the production function of seminars/tutorials is $F(n) = 30000n - 62.5n^2$, n is the number of GTAs hired. The inverse supply function of GTAs is given by $W(n) = 1000 + 75n$ where W is the wage (as an annual salary) that must be paid so n GTAs are willing to work at that wage. You can assume that the price at which the university can "sell" seminars/tutorials is just 1.

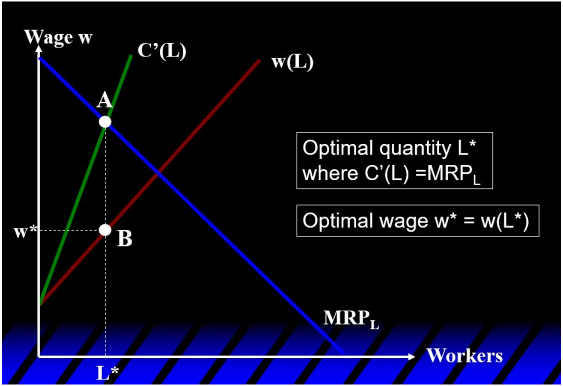
a) If the Central London universities, behaving as a single entity, take advantage of its monopsonit position, how many GTAs will be hired? What wage will be paid?

We know that a monopsony will maximise profits when (slide 38-40, lecture 13):

$$MRP_L = C'(L) = p \cdot MP_L \tag{57}$$

$$W^* = w(L^*) \tag{58}$$

The firm hires workers until the cost of an additional worker equals the revenue generated by the additional units produced thanks to that worker


Wage w
C'(L)
w(L)
A
Optimal quantity L*
where C'(L) =MRP_L
Optimal wage w* = w(L*)
w*
B
MRP_L
Workers
L*

How we can get the total cost function and then, the marginal cost function?

We know that the (total) cost function is equal to the supply function × the quantity, thus:

$$TC = \text{supply function} \times \text{quantity} \tag{59}$$

$$TC(n) = W(n) \cdot n \Longrightarrow (1000 + 75n) \cdot n \tag{60}$$

$$TC(n) = 1000n + 75n^2 \tag{61}$$

Then, our marginal cost is the derivative wrt n:

$$\frac{\partial TC(n)}{\partial n} = MC = 1000 + 2 \cdot 75n \tag{62}$$

$$MC = 1000 + 150n \tag{63}$$

As a monopsonist, the Central Universities will hire GTAs as long as the MR is equal to MC, but let's first get the MR:

$$F(n) = 30000n - 62.5n^2 \tag{64}$$

Then, the MR is equal to:

$$MR = MP \cdot 1 \Longrightarrow 30000 - 125n \tag{65}$$

$$MR = MC \Longrightarrow 30000 - 125n = 1000 + 150n \tag{66}$$

$$n^* = 105.45 \tag{67}$$

What wage will be paid?

The monopsonist firm will pay:

$$w^* = w(L^*) \tag{68}$$

Substituting this into the supply curve to determine the wage:

$$w(n) = 1000 + 75n \Longrightarrow 1000 + 75 \cdot 105.45 \tag{69}$$

$$w(n)^* = £8,908.75 \tag{70}$$

If the government sets a minimum wage level of £10,000 for GTAs, how many GTAs would be hired?

With a minimum wage, the marginal cost of hiring one GTA is £10,000, and we still need to meet this condition:

$$MRP_L = C'(L) = p \cdot MP_L \tag{71}$$

$$30000 - 125n = 10000 \tag{72}$$

$$n^* = 160 \tag{73}$$

This is how many workers the firm (universities) would be willing to hire at that wage. However, at teh minimum wage only

$$w(n) = 1000 + 75n \Longrightarrow 1000 + 75 \cdot n \tag{74}$$

$$n = 120 \text{ workers would be willing to work} \tag{75}$$

Question 4.a

a) A monopolist is deciding how to allocate output between two geographically separated markets (London and Paris). Demand in the two markets is $P_1 = 15 - Q_1$ in London and $P_2 = 25 - 2Q_2$ in Paris. The monopolist's total cost is $C(Q_1 + Q_2) = 5 + 3(Q_1 + Q_2)$. What are the price(s), total output, profit, and total dead-weight loss:

a. What are the price(s), total output, profit, and total dead-weight loss:

Is this a first, second, or third price discrimination problem?

Is this a first, second, or third price discrimination problem?

Definition: A policy of third-degree price discrimination offers a different price for each segment of the market (or each consumer group) when membership in a segment can be observed (slide 31, lecture 14).

The monopoly chooses a quantity in each market such that marginal revenue is equal to the marginal cost in each market. Stated mathematically (see slide 38, lecture 14):

$$\frac{\partial(p_1(y_1)y_1)}{\partial y_1} = \frac{\partial(p_2(y_2)y_2}{\partial y_2)} = \frac{\partial c(y_1 + y_2)}{\partial(y_1 + y_2)} \tag{76}$$

$$MR_1(y_1) = MR_2(y_2) = MC(y1, y2) \tag{77}$$

Let's get the MRs and MC. The MR for market 1 is:

$$P_1 = 15 - Q_1 \tag{78}$$

$$MR_1 = 15 - 2Q_1 \tag{79}$$

MR for market 2:

$$P_2 = 25 - 2Q_2 \tag{80}$$

$$MR_2 = 25 - 4Q_2 \tag{81}$$

Our MC is equal to:

$$C(Q_1 + Q_2) = 5 + 3(Q_1 + Q_2) \tag{82}$$

$$MC = \frac{\partial C(Q_1 + Q_2)}{\partial (Q_1 + Q_2)} = \cancelto{0}{5} + \frac{3\partial (Q_1 + Q_2)}{\partial (Q_1 + Q_2)} \tag{83}$$

$$MC = 3 \tag{84}$$

Thus, we equate the MRs=MC:

$$MR_1 = MC \implies 15 - 2Q_1 = 3 \implies Q_1 = 6 \text{ in London} \tag{85}$$

$$MR_2 = MC \implies 25 - 4Q_2 = 3 \implies Q_2 = 5.5 \text{ in Paris} \tag{86}$$

In order to find the prices in each market, we substitute each quantity into the respective demand functions. For market 1 (London):

$$P_1 = 15 - Q_1 \implies P_1 = 15 - 6 \implies P_1^* = £9 \tag{87}$$

For market 2 (Paris):

$$P_2 = 25 - 2Q_2 \implies P_2 = 25 - 2 \cdot 5.5 \implies P_2^* = £14 \tag{88}$$

Just to remind you (and me), they asked us:

What are the price(s), total output, profit, and total dead-weight-loss.

Total output is equal to the sum of outputs in each market:

$$Q = Q_1 + Q_2 = 6 + 5.5 \tag{89}$$

$$Q^* = 11.5 \tag{90}$$

The total profit function is equal to:

$$\pi(y_1, y_2) = p_1(y_1)y_1 + p_2(y_2)y_2 - c(y_1 + y_2) \tag{91}$$

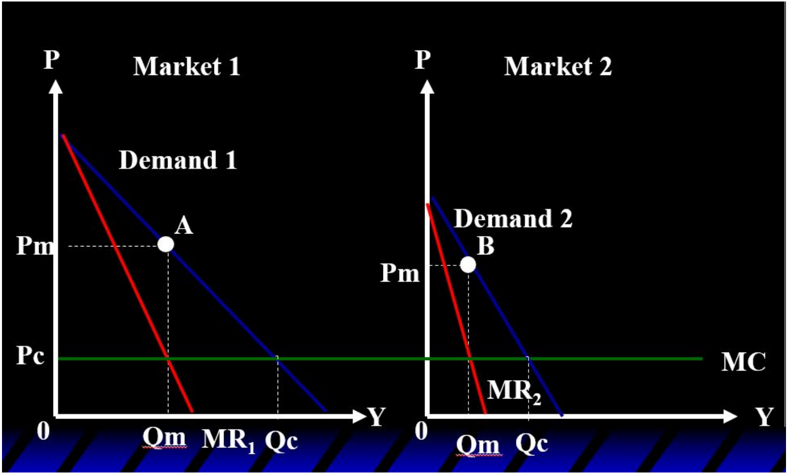
$$\pi(Q_1, Q_2) = £9 \cdot 6 + £14 \cdot 5.5 - (5 + 3 \cdot (11.5)) \tag{92}$$

$$\pi(Q_1, Q_2) = £91.5 \tag{93}$$

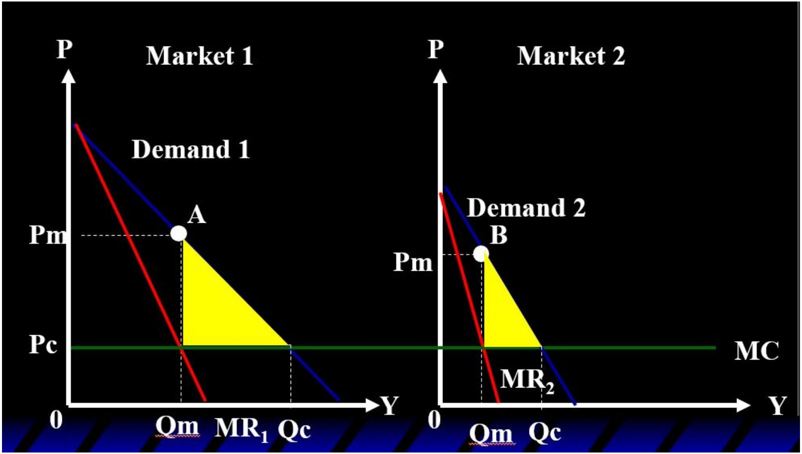
Let's get the dead-weight loss:

$$DWL = \frac{(Q_c - Q_m)(P_m - P_c)}{2} \tag{94}$$

Let's see this in the price-quantity demand graph:



Let's see this in the price-quantity demand graph:



What is the market price?

$MC = P_c = £3$, thus the demand in each market when $p = 3$ is. For market 1:

$$P_1 = 15 - Q_1 \implies 3 = 15 - Q_1 \implies Q_1 = 12 \tag{95}$$

Q_c for market 2:

$$P_2 = 25 - 2Q_2 \implies 3 = 25 - 2Q_2 \implies Q_2 = 11 \tag{96}$$

Let's calculate the DWLs for each market:

$$DWL_1 = \frac{(Q_c - Q_m)(P_m - P_c)}{2} \implies DWL_1 = \frac{(12-6)(9-3)}{2} \tag{97}$$

$$DWL_1 = 18 \tag{98}$$

$$DWL_2 = \frac{(Q_c - Q_m)(P_m - P_c)}{2} \implies DWL_1 = \frac{(11-5.5)(14-3)}{2} \tag{99}$$

$$DWL_2 = 30.25 \tag{100}$$

$$DWL_t = 18 + 30.25 \implies 48.25 \tag{101}$$

Question 3.b

b. If the law prohibits charging different prices in the two regions?

Without price discrimination the monopolist must charge a single price for the entire market. To maximise profit, find the quantity such that marginal revenue is equal to marginal cost. Adding demand equations, we find that the total demand curve has a kink at P = 15. This implies the aggregate demand equations is equal to:

$$P = \begin{cases} 25 - 2Q \text{ if } Q \leq 5 \\ 18.33 - 0.67Q \text{ if } Q > 5 \end{cases} \tag{102}$$

$$MR = \begin{cases} 25 - 4Q \text{ if } Q \leq 5 \\ 18.33 - 1.33Q \text{ if } Q > 5 \end{cases} \tag{103}$$

How do we aggregate the demands above $Q > 5$:

Step 1: Rearrange the demand functions as functions of Q

$$P_1 = 15 - Q_1 \implies Q_1 = 15 - P_1 \tag{104}$$

$$P_2 = 25 - 2Q_2 \implies 2Q_2 = 25 - P_2 \implies Q_2 = 12.5 - \frac{P_2}{2} \tag{105}$$

Add the demand functions:

$$Q_1 + Q_2 = 15 - P_1 + 12.5 - \frac{P_2}{2} \implies 15 + 12.5 - P_1 - \frac{P_2}{2} \tag{106}$$

Add the demand functions:

$$Q_1 + Q_2 = 15 - P_1 + 12.5 - P_2 \Longrightarrow 15 + 12.5 - P_1 - \frac{P_2}{2} \tag{107}$$

$$Q = 27.5 - P - \frac{P}{2} \Longrightarrow Q = \frac{55 - 3P}{2} \tag{108}$$

$$Q = \frac{55 - 3P}{2} \Longrightarrow 2Q = 55 - 3P \Longrightarrow 3P = 55 - 2Q \tag{109}$$

$$3P = 55 - 2Q \Longrightarrow P = 18.33 - 0.67Q \tag{110}$$

So this is how we get the aggregate demand for $Q > 5$

The MC is equal to 3, thus we can use the demand function under $Q > 5$ where we need to find the quantity demanded when MR = MC:

$$MR = MC \implies 18.33 - 1.33Q = 3 \implies Q = 11.5 \tag{111}$$

Then, the price at that quantity demanded is equal to:

$$P = 18.33 - 0.67Q \implies P = 18.33 - 0.67 \cdot 11.5 \tag{112}$$

$$P^{*} = 10.67 \tag{113}$$

Market 1:

$$10.67 = 15 - Q_1 \implies Q_1 = 4.33 \tag{114}$$

Market 2:

$$10.67 = 25 - 2Q_2 \implies Q_2 = 7.17 \tag{115}$$

Now that we have the market price, quantities, and cost function we can get the new profits:

$$\pi(y_1, y_2) = p_1(y_1)y_1 + p_2(y_2)y_2 - c(y_1 + y_2) \tag{116}$$

$$\pi(Q_1, Q_2) = £10.67 \cdot 4.33 + £10.67 \cdot 7.17 - (5 + 3 \cdot (11.5)) \tag{117}$$

$$\pi(Q_1, Q_2) = £83.21 \tag{118}$$

Finally, the dead-weight loss in each market is equal to:

$$DWL_1 = \frac{(Q_c - Q_m)(P_m - P_c)}{2} = \frac{(12 - 4.33)(10.67 - 3)}{2} \tag{119}$$

$$DWL_1 = 29.41 \tag{120}$$

$$DWL_2 = \frac{(Q_c - Q_m)(P_m - P_c)}{2} = \frac{(11 - 7.17)(10.67 - 3)}{2} \tag{121}$$

$$DWL_2 = 14.69 \tag{122}$$

$$DWL_t = 29.41 + 14.69 \Longrightarrow 44.10 \tag{123}$$

Total dead-weight loss is 44.10. Without price discrimination, profit is lower, dead-weight loss is lower, and total output is unchanged. The big winners are consumers in market 2 who now pay £10.67 instead of £14. DWL in market 2 drops from 30.25 to 14.69. Consumers in market 1 and the monopolist are worse off when price discrimination is not allowed.

Question 5.a

Sal's satellite company broadcasts TV to subscribers in England and Wales. The demand functions for each of these two regions are: $Q_E = 60–0.25P_E$ and $Q_W = 100–0.50P_W$ where Q is in thousands of subscriptions per year and P is the subscription price per year. The cost of providing Q units of service is given by $C(Q) = 1000 + 40Q$ where $Q = Q_E + Q_W$.

What are the profit-maximising prices and quantities for the English and Welsh markets?

Are we in a first, second or third price discrimination problem?

Again, we are in a third price discrimination problem.

Sal should pick quantities in each market so that the marginal revenues are equal to one another and equal to marginal cost. To determine marginal revenues in each market, first solve for price as a function of quantity:

$$\frac{\partial(p_E(y_E)y_E)}{\partial y_E} = \frac{\partial(p_W(y_W)y_W)}{\partial y_W} = \frac{\partial c(y_E + y_W)}{\partial(y_E + y_W)} \tag{124}$$

$$MR_E(y_E) = MR_W(y_W) \tag{125}$$

$$P_E = 240 - 4Q_E \tag{126}$$

$$P_W = 200 - 2Q_W \tag{127}$$

Then, marginal revenue is equal to:

$$MR_E = 240 - 8Q_E \tag{128}$$

$$MR_W = 200 - 4Q_W \tag{129}$$

Set each MR equal to MC, which is £40, and determine the profit-maximising quantity in each submarket:

$$MR_E = 240 - 8Q_E \implies 240 - 8Q_E = 40 \implies Q_E^* = 25 \tag{130}$$

$$MR_W = 200 - 4Q_W \implies 200 - 4Q_W = 40 \implies Q_W^* = 40 \tag{131}$$

Next, to determine the price in each submarket, we substitute the profit-maximising quantity into the respective demand equation:

$$P_E = 240 - 4Q_E \implies P_E = 240 - 4 \cdot 25 \tag{132}$$

$$P_W = 200 - 2Q_W \implies P_W = 200 - 2 \cdot 40 \tag{133}$$

$$P_E^* = 140 \tag{134}$$

$$P_W^* = 120 \tag{135}$$

Question 5.b

b. As a consequence of the recent launch of a new satellite, people in Wales can receive Sal's England broadcasts, and people in England can receive Sal's Wales broadcasts. As a result, anyone in England or Wales can receive Sal's broadcasts by subscribing in either country. Thus, Sal can charge only a single price. What price should he charge, and what quantities will he sell in each region?

Now, Sal faces an aggregate demand function, thus, we need to compute this combined demand.

Sal's combined demand function is the horizontal summation of the E and W demand functions. Above a price of £200[1] (the vertical intercept of the W demand function), the total demand is just the England demand function, whereas below a price of £200, we add the two demands. One way to get the combined demand is to the following:

$$P_E = 240 - 4Q_E \Longrightarrow Q_E = \frac{240}{4} - \frac{1P_E}{4} \Longrightarrow Q_E = 60 - 0.25P_E \quad (136)$$

$$P_W = 200 - 2Q_W \Longrightarrow Q_w = \frac{200}{4} - \frac{2P_w}{4} \Longrightarrow Q_w = 50 - 0.5P_W \quad (137)$$

[1] Prices between £240 and £200, there is only demand from England

Then, we aggregate the quantities and solve for P as a function of Q:

$$Q_T = 60 - 0.25P + 50 - 0.5P = 160 - 0.75P \tag{138}$$

$$P = 213.33 - 1.333Q_T \tag{139}$$

Then, the MR is equal to:

$$MR = 213.33 - 2.667Q_T \tag{140}$$

If we set MR = MC, and our MC = £40

$$MR = MC \implies 213.33 - 2.667Q_T = 40 \implies Q_T = 65 \tag{141}$$

We substitute $Q_T = 65$ into the inverse demand function to determine the price:

$$P = 213.33 - 1.333(65) \implies P = 126.67 \tag{142}$$

Although a price of £126.67 is charged in both markets, different quantities are purchased in each market:

$$Q_E = 60 - 0.25P_E \implies Q_E = 60 - 0.25 \cdot 126.67 \implies Q_E = 28.3 \tag{143}$$

$$Q_w = 50 - 0.5P_W \implies Q_w = 50 - 0.5 \cdot 126.67 \implies Q_W = 36.7 \tag{144}$$

Question 5.c

c. In which of the above situations, (a) or (b), is Sal better off? In terms of consumer surplus, which situation do people in England prefer and which do people in Wales prefer?

How would you determine in which situation Sal would be better off?

We need to compare profits in both situations.

$$\pi(y_1, y_2) = p_1(y_1)y_1 + p_2(y_2)y_2 - c(y_1 + y_2) \tag{145}$$

For situation 1, where Sal can discriminate prices:

- $P_E = 140$
- $P_W = 120$
- $Q_E = 25$
- $Q_w = 40$

For situation 2, where Sal **CANNOT** discriminate prices:

- $P_E = 126.67$
- $Q_T = 65$

Sal's cost function $C(Q) = 1000 + 40Q$

For situation 1, where Sal **CAN** discriminate prices:

- $P_E = 140$
- $P_W = 120$
- $Q_E = 25$
- $Q_w = 40$

$$\pi(Q_1, Q_2) = 140 \cdot 25 + 120 \cdot 40 - (1000 + 40(65)) \tag{146}$$

$$\pi(Q_1, Q_2) = £4700 \tag{147}$$

For situation 2, where Sal **CANNOT** discriminate prices:

- $P_E = 126.67$
- $Q_T = 65$

$$\pi(Q) = 126.67 \cdot 65 - (1000 + 40(65)) \tag{148}$$

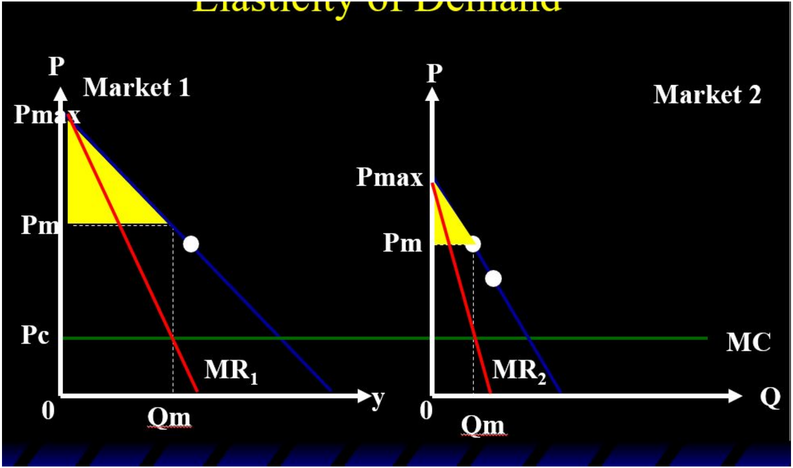
$$\pi(Q) = £4633.33 \tag{149}$$

Therefore , Sal is better off when the two markets are separate.

In terms of consumer surplus, which situation do people in England prefer and which do people in Wales prefer?

How would you compare which situation people from England and Wales prefer?

You need to calculate and compare the consumer surplus


P
Market 1
Pmax
Pm
Pc
MR1
0
Qm
y
P
Market 2
Pmax
Pm
MC
MR2
0
Qm
Q

$$CS = Q_m \frac{P_m ax - P_m}{2} \tag{150}$$

You can get Pmax setting Q equal to zero in the inverse demand function. In scenario 1:

$$CS_E = (25)\frac{240 - 140}{2} = 1,250 \tag{151}$$

$$CS_W = (40)\frac{200 - 120}{2} = 1,600 \tag{152}$$

Without price discrimination:

$$CS_E = (28.3)\frac{240 - 126.67}{2} = 1,603.67 \tag{153}$$

$$CS_W = (36.7)\frac{200 - 126.67}{2} = 1,345.67 \tag{154}$$

English people would prefer b. because their price is £126.67 instead of £140, giving them higher consumer surplus. Customers in Wales prefer a because their price is £120 instead of £126.67, and their consumer surplus is greater.

Question 6.a

6. As the owner of the only tennis club in an isolated wealthy community, you must decide on membership dues (T) and fees for court time (P). There are two types of tennis players. "serious" players have demand $Q_1 = 10 – P$. There are also "occasional" players with demand: $Q_2 = 4 – 0.25P$ where Q is court hours per week and P is the fee per hour for each individual player. Assume that there are 1000 players of each type. Because you have plenty of courts, the marginal cost of court time is zero. You have fixed costs of £10,000 per week. Serious and occasional players look alike and you cannot tell them apart, so you must charge them the same fees for court time and membership dues.

a. Suppose that to maintain a "professional" atmosphere, you want to limit membership to serious players. How should you set the weekly membership dues (T) and court fees (P) to maximise profits, keeping in mind the constraint that only serious players choose to join? What would profits be per week?

Which type of price-discrimination problem is this one?

We are under a second price discrimination problem.

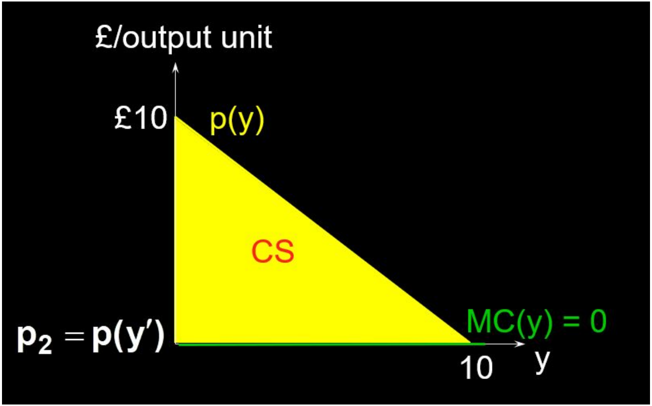
In order to limit membership to serious players, the club owner should charge an entry fee, T, equal to the total consumer surplus of serious players and a usage fee P equal to marginal cost of zero. With individual demands of $Q1 = 10 - P$, individual consumer surplus is equal to:

Let's remember that in a second price discrimination price, the monopoly will charge a entry fee T = CS, and a variable fee $p_2 = MC$

$$Q_1 = 10 - P \implies 0 = 10 - P_{max} \implies P_{max} = 10 \tag{155}$$

$$C(Q) = 10,000 \implies MC = 0 \tag{156}$$

What is the consumer surplus (CS) then?



Then, the customer surplus and T should be equal to:

$$CS = \frac{(10-0)(10-0)}{2} = £50 \text{ per week} \tag{157}$$

An entry fee of £50 maximises profits by capturing all consumer surplus. The profit-maximising court fee is set to zero, because marginal cost is equal to zero. The entry fee of T= £50 is higher than the occasional players are willing to pay (higher than their consumer surplus at a court fee of zero); therefore, this strategy will limit membership to the serious players.

They tell us that the are 1000 "serious" players

$$\pi = (£50)(1000) - £10000 = £40,000 \tag{158}$$

Question 6.b

b. A friend tells you that you could make greater profits by encouraging both types of players to join. What weekly membership dues (T) and court/hour fees (P) would maximise weekly profits? What would these profits be (and is your friend right)?

How to approach this question?

- Given that the firm has fixed costs what the firm needs to is maximise its revenue. The higher the revenue the higher the profits.
- The first step is to set the entry fee that both consumer will be willing to pay.
- This entry fee will be equal to the consumer surplus with the lesser demand (occasional players).
- Rather than setting the price equal to the marginal cost, the owner can set up a higher price in order to capture both consumers' surplus and increase his profits.
- Thus, we need to find the price P that 1) it sets a entry fee that both consumers will be willing to pay 2) Captures both consumers' surplus.
- And we will find this using the profit function Total profits: CS + court fee - fixed costs

Question 6.b

b. When there are two classes of customers, serious and occasional players, the club owner maximises profits by charging court fees above marginal cost and by setting the entry fee (weekly dues) **T equal to the remaining consumer surplus of the consumer with the lesser demand**, in this case, the occasional player. The entry fee, T, equals the consumer surplus remaining after the court fee P is assessed:

$$T = CS_{Occasional} = \frac{(Q_2)(16 - P)}{2} \tag{159}$$

Where:

- Q_2 is the quantity demanded when the $MC = P$
- Max price for "occasional players" $Q_2 = 4–0.25P$, then $P_{max} = 16$
- $Q_2 = 4–0.25P$

The first set is set a price that determines an entry fee that both consumers will be willing to pay:

$$T_{Occasional} = \frac{(Q_2)(16 - P)}{2} \implies \frac{(4 - 0.25P)(16 - P)}{2} \tag{160}$$

This is the entry fee:

$$T_{Occasional} = 0.5(4 - 0.25P)(16 - P) \implies 32 - 4P + 0.125P^2 \tag{161}$$

Then, the total entry fee paid for all players is:

$$T \times \text{N of players} \tag{162}$$

$$T \cdot 2000 = 2000 \cdot (32 - 4P + 0.125P^2) \tag{163}$$

$$T \cdot 2000 = 64000 - 8000P + 250P^2 \tag{164}$$

The function above represents the entry fee, which is a function of P, which makes sense as we will only charge a fee that is equal to the consumer surplus. And the consumer surplus depends on the price.

Now, let's calculate the revenue from court fees:

$$TR = P \times Q \tag{165}$$

In this case:

$$TR_{courtfee} = P \times Q \Longrightarrow P(Q_1 + Q_2) = P[1000 \cdot (10 - P) + 1000 \cdot (4 - 0.25P)] \tag{166}$$

$$TR_{courtfee} = 14000P - 1250P^2 \tag{167}$$

Here again the court fees will depend on the quantity demanded, which is a function of P.

Then, total revenue is equal to:

$$TR = TR_{entryfee} + TR_{courtfee} \tag{168}$$

$$TR = 64000 - 8000P + 250P^2 + 14000P - 1250P^2 \tag{169}$$

$$TR = 64000 + 6000P - 1000P^2 \tag{170}$$

Our marginal cost is equal to zero, so we want to maximise total revenue.

$$C(Q) = 10,000 \implies MC = 0 \tag{171}$$

To do this, differentiate total revenue with respect to price and set the derivative to zero:

$$TR = 64000 + 6000P - 1000P^2 \tag{172}$$

We derive TR with respect to P:

$$\frac{\partial TR}{\partial P} = \cancel{\frac{\partial 64000}{\partial P}}^{0} + \frac{\partial 6000P}{\partial P} - \frac{\partial 1000P^2}{\partial P} = 0 \tag{173}$$

$$\frac{\partial TR}{\partial P} = 6000 - 2 \cdot 1000^{2-1} \Longrightarrow 6000 - 2000P = 0 \tag{174}$$

$$P = £3 \text{ Solving for the optimal court fee, P} = £3 \text{ per hour.} \tag{175}$$

What weekly membership dues (T) and court/hour fees (P) would maximise weekly profits? What would these profits be (and is your friend right)?

Membership:

$$T = 32 - 4P + 0.125P^2 \implies 32 - 4 \cdot 3 + 0.125 \cdot 3^2 \tag{176}$$

$$T = £21.125 \tag{177}$$

and court/hour fees:

$$TR_{courtfee} = 14000P - 1250P^2 \implies 14000 \cdot 3 - 1250 \cdot 3^2 \tag{178}$$

$$TR_{courtfee} = £30,750 \tag{179}$$

Total revenue:

$$TR = 64000 + 6000P - 1000P^2 \implies TR = 64000 + 6000 \cdot 3 - 1000 \cdot 3^2 \quad (180)$$

$$TR = £73,000 \quad (181)$$

Total profit:

$$\pi = TR - TC = 73,000 - 10,000 = £63,000 \text{ per week} \quad (182)$$

$$\pi = £63,000 \text{ per week} \quad (183)$$

Which is greater than the £40,000 profit when only serious players become members. Therefore, your friend is right; it is more profitable to encourage both types of players to join.

Office hours: Fridays from 11:00 to 12:00 hrs.
Bush House (NE) 9.01

Microeconomics (5SSPP217)

Seminar Problem Set 8

Felipe Torres
felipe.torres@kcl.ac.uk

June 4, 2025

Question 1.a

1. There are two firms in the market, each of which produce at a constant average (and marginal) cost of AC = MC = £5. Let Q1 be the output of the first firm and Q2 be the output of the second. Market demand is given by Q = 52 - P, where Q = Q1 + Q2.

a. Suppose (as in the Cournot model) that each firm chooses its profit-maximising level of output on the assumption that its competitor's output is fixed. Find each firm' reaction curve.

Under the Cournot model, each firm treats the output of the other firm(s) as a constant in its maximisation calculations. Therefore, Firm 1 chooses Q_1 to maximise π_1 with Q_2 being treated as a constant.

We know that Q is equal to:

$$Q = Q_1 + Q_2 \tag{1}$$

Then, Firm 1's profit function is:

$$\pi_1 = P \cdot Q_1 - MC \cdot Q_1 \text{ and } Q = 52 - P \Longrightarrow Q_1 + Q_2 = 52 - P \tag{2}$$

$$\pi_1 = Q_1(52 - Q_1 - Q_2) - 5Q_1 \Longrightarrow \pi_1 = 52Q_1 - Q_1^2 - Q_1Q_2 - 5Q_1 \tag{3}$$

$$\pi_1 = 47Q_1 - Q_1^2 - Q_1Q_2 \tag{4}$$

Deriving w.r.t Q_1 we will find the profit maximising quantity for firm 1.

$$\frac{\partial \pi_1}{\partial Q_1} = \frac{\partial 47 Q_1}{\partial Q_1} - \frac{\partial Q_1^2}{\partial Q_1} - \frac{\partial Q_1 Q_2}{\partial Q_1} = 0 \qquad (5)$$

$$\frac{\partial \pi_1}{\partial Q_1} = 47 - 2Q_1 - Q_2 \Longrightarrow Q_1 = \frac{47 - Q_2}{2} \qquad (6)$$

where the last equation is the reaction (or best response) function for Firm 1 which gives its the profit-maximising level of output, given the output of Firm 2:

$$Q_1 = R_1(Q_2) = \frac{47 - Q_2}{2} \qquad (7)$$

Because the problem is symmetric, i.e. identical costs for both firms, the reaction function for Firm 2 is:

$$Q_2 = R_2(Q_1) = \frac{47 - Q_1}{2} \tag{8}$$

Question 1.b

b. Calculate the Cournot equilibrium quantities. What are the resulting market price and profit of each firm?

Solve for the values of Q_1 and Q_2 that satisfy both reaction functions by substituting, for instance, Firm 2s reaction function into the function for Firm 1:

You could do what is in slide 32, lecture 15, which is substituting Q_1 into the BR function of Q_2.

$$Q_2 = R_2(Q_1) = \frac{47 - Q_1}{2} \tag{9}$$

$$Q_1 = R_1(Q_2) = \frac{47 - Q_2}{2} \Longrightarrow Q_1 = \frac{47 - \frac{(47-Q_1)}{2}}{2} \tag{10}$$

$$Q_1 = \frac{94 - 47 + Q_1}{4} \Longrightarrow 4Q_1 = 94 - 47 + Q_1 \tag{11}$$

$$Q_1^* = \frac{47}{3} = 15.67 \tag{12}$$

By symmetry $Q_2^* = 15.67$

To determine the price, substitute Q_1^* and Q_2^* (Equal to Q) into the demand equation:

$$Q = 52 - P \tag{13}$$

$$P^* = 52 - \left(\frac{47}{3} + \frac{47}{3}\right) = \frac{62}{3} = £20.67 \tag{14}$$

Profit for Firm 1 is therefore:

$$\pi_1^* = P^* Q_1^* - C(Q_1^*) \tag{15}$$

$$\pi * = P^* Q_1^* - C(Q_1^*) \tag{16}$$

$$\pi * = 20.67 \cdot 15.67 - 5 \cdot 15.67 = £245.45 \tag{17}$$

By symmetry, firm's 2 profits are: £245.45

Question 1.c

c. Suppose instead there are N firms in the industry, all with the same constant marginal cost of £5. Find the reaction function of one of the N firms and the Cournot equilibrium quantity it will produce as a function of N. (Note that now $Q = Q_1 + Q_2 + ... + Q_N$).

If there are N identical firms, then the price in the market will be $P = 52 - \underbrace{(Q_1 + Q_2 + \ldots + Q_i + \ldots + Q_N)}_{\mathbf{Q}}$.

$$\pi_i = PQ_i - C(Q_i) \tag{18}$$

$$\pi_i = (52 - (Q_1 + Q_2 + \ldots + Q_i + \ldots + Q_N)) \cdot Q_i - MC \cdot Q_i \tag{19}$$

$$\pi_i = 52Q_i - (Q_1 Q_i + Q_2 Q_i + Q_i^2 \ldots + Q_N Q_i) - 5Q_i \tag{20}$$

Differentiate to obtain the first order condition:

$$\frac{\partial \pi_i}{\partial Q_i} = 52 - Q_1 - Q_2 - \cdots - 2Q_i - \ldots - Q_N - 5 = 0. \tag{21}$$

Solving for Q_i

$$2Q_i = 52 - 5 - (Q_1 + Q_2 + \cdots + Q_{i-1} \ldots Q_{i+1} + \ldots + Q_N) \tag{22}$$

$$Q_i = \frac{47}{2} - \frac{1}{2}(Q_1 + \ldots + Q_{i-1} + Q_{i+1} + \ldots + Q_N) \tag{23}$$

As all firms face the same costs, they will all produce the same level of output; therefore $Q_i = Q^*, Q_{i-1} = Q^*, Q_{i+1} = Q^*$. Substituting all firms' quantities by Q^*:

$$Q^* = \frac{47}{2} - \frac{1}{2}(N-1)Q^* \implies 2Q^* = 47 - (N-1)Q^* \tag{24}$$

$$2Q^* = 47 - NQ* + Q^* \implies Q^* + NQ^* = 47 \tag{25}$$

$$Q^* + NQ^* = 47 \implies Q^*(1+N) = 47 \tag{26}$$

$$Q^* = \frac{47}{N+1} \tag{27}$$

This is the individual level of output, thus we need to multiply by the number (N) of firms.

This is the individual level of output, thus we need to multiply by the number (N) of firms.

$$\mathbf{Q} = Q^* \cdot N = \frac{\mathbf{N} \cdot 47}{N+1} \implies \mathbf{Q} = \frac{N \cdot 47}{N+1} \tag{28}$$

$$\mathbf{Q} = \frac{47N}{N+1} \tag{29}$$

Question 1.d

d. What will the market price and the total profit of all N firms be (also as functions of N)?

Now substitute $\mathbf{Q} = N \cdot Q^* = \frac{47N}{N+1}$ for total output in the demand function:

$$Q = 52 - P \tag{30}$$

$$P^* = 52 - (\frac{47N}{N+1}) \tag{31}$$

$$P^* = \frac{5N + 52}{N+1} \tag{32}$$

Total profits are:

$$\pi_T = P^* \mathbf{Q} - C(\mathbf{Q}) \tag{33}$$

$$\pi_T = \frac{5N + 52}{N + 1} \cdot \frac{47N}{N + 1} - 5 \cdot \frac{47N}{N + 1} \tag{34}$$

$$\pi_T = P\mathbf{Q} - 5\mathbf{Q} \Longrightarrow 47N \frac{5N + 52}{(N + 1)^2} - \frac{235N}{N + 1} \tag{35}$$

$$\pi_T = \frac{2209}{(N + 1)^2} \tag{36}$$

Question 1.e

What do the market output, price and total profit tend towards as N becomes large?

Taking the limit $N \longrightarrow \infty$ yields $\mathbf{Q} = 47$ and $P^* = 5$ (which is the MC) and $\pi_T = 0$. Thus, when N approaches infinity, this market approaches a perfectly competitive one.

$$P^* = \frac{5N + 52}{N + 1} \tag{37}$$

$$P^* = 5\underbrace{\frac{N}{\cancel{N} + 1}}_{\to 1} + \underbrace{\frac{52}{\cancel{N} + 1}}_{\to 0} \tag{38}$$

Then, we plug in the price into the demand function:

$$Q = 52 - P \Longrightarrow Q = 52 - 5 \tag{39}$$

$$Q = 47 \tag{40}$$

Quetion 1.f

f. Go back now to the two-firms case but suppose now that Firm 1 is the Stackelberg leader. How much will each firm produce and what will their profits be?

We know that in the Stackelberg leader theory, the leader can observe the follower's reaction curve.

Firm 2's reaction curve is the same as determined in part $(a) : Q2 = \frac{47-Q_1}{2}$. When Firm 1 makes its choice, it knows how Firm 2 will react. In other words, Firm 1 uses its knowledge of Firm 2's reaction function when determining its optimal output.

Hence, Firm 1, the Stackelberg leader, will choose its output, Q1, to maximize its profits, subject to Firm 2 using its reaction function. In other words, Firm 1 maximises:

In other words, Firm 1 maximises:

$$\pi_1 = P(Q_1 + Q_2(Q)1) \cdot Q_1 - C(Q_1) \tag{41}$$

$$\pi_1 = (52 - Q_1 - \frac{(47 - Q_1)}{2}) \cdot Q_1 - 5Q_1 \tag{42}$$

To determine the profit-maximising quantity, take the derivative and set it equal to 0:

$$\frac{\partial \pi_1}{\partial Q_1} = 52 - 2Q_1 - (\frac{47}{2}) + Q_1 - 5 = 0 \tag{43}$$

$$Q_1 = 23.5 \tag{44}$$

Substituting Q1 = 23.5 into Firm 2's reaction function gives Q2 = 11.75.

$$Q_1 = 23.5 \tag{45}$$

$$Q_2 = R(Q_1) = \frac{47 - Q_1}{2} \implies Q_2 = \frac{47 - 23.5}{2} \tag{46}$$

$$Q_2 = 11.75 \tag{47}$$

Next, substitute Q_1 and Q_2 into the demand equation to find the price ($Q = Q_1 + Q_2$):

$$Q = 52 - P \Longrightarrow 23.5 + 11.75 = 52 - P \tag{48}$$

$$P = 52–23.5–11.75 = \pounds 16.75. \tag{49}$$

$$P = \pounds 16.75. \tag{50}$$

Profits for each firm are equal to total revenue minus total costs:

$$\pi_1 = 16.75 \cdot 23.5 - 5 \cdot 23.5 = \pounds 276.125 \tag{51}$$

$$\pi_2 = 16.75 \cdot 11.75 - 5 \cdot 11.75 = \pounds 138.062 \tag{52}$$

Total industry profit is $\pi_1 + \pi_2 = \pounds 414.19$. Compared to the Cournot equilibrium, total output has increased from 31.34 to 35.25, price has fallen from £20.67 to £16.75, and total profits have fallen from £490.89 to £414.1875. Profits for Firm 1 have risen from £245.45 to £276.125, while the profits for Firm 2 have declined sharply from £245.45 to £138.0625.

Question 2.a

Suppose that duopoly firms have constant marginal costs of £10 per unit but produce horizontally differentiated products. Firm 1 faces a demand function of $Q_1 = 100 - 2P_1 + P_2$ and firm 2 faces a demand of $Q_2 = 100 - 2P_2 + P_1$.

a. Are the two goods substitutes or complements?

We did this in seminar 1.

We need to calculate the derivative of the demand function of Q_1 with respect to price of good 2:

$$Q_1 = 100 - 2P_1 + P_2 \tag{53}$$

$$\frac{\partial Q_1}{\partial P_2} = \frac{\partial 100}{\partial P_2} - \frac{\partial 2P_1}{\partial P_2} + \frac{\partial P_2}{\partial P_2} \tag{54}$$

$$\frac{\partial Q_1}{\partial P_2} = 1 \tag{55}$$

$$\frac{\partial Q_1}{\partial P_2} > 0 \tag{56}$$

The demand of good 1 is increasing in the price of good 2, and vice-versa. Whenever the price of one good increases, consumers switch to buying the other good. Therefore, the goods are substitutes.

Question 2.b

b. Derive the best response functions and solve for the Bertrand equilibrium prices, quantities and profits.

Firm 1 maximise its profit:

$$\max \pi_1 = P_1 Q_1(P_1, P_2) - MCQ_1(P_1, P_2) \tag{57}$$

$$\pi_1 = P_1 Q_1(P_1, P_2) - 10 \cdot Q_1(P_1, P_2) \tag{58}$$

$$\pi_1 = (P_1 - 10) Q_1 = (P_1 - 10)(100 - 2P_1 + P_2) \tag{59}$$

It's first order condition is:

$$\frac{\partial \pi_1}{\partial P_1} = 100 - 4P_1 + P_2 + 20 = 0 \tag{60}$$

So firm 1's best response function is:

$$P_1 = 30 + \frac{P_2}{4} \tag{61}$$

Similarly, firm 2's best response function is:

$$P_2 = 30 + \frac{P_1}{4} \tag{62}$$

Substituting one into the other we find the equilibrium prices:

$$P_2 = 30 + \frac{30 + \frac{P_2}{4}}{4} \Longrightarrow P_2 = 30 + \frac{30}{4} + \frac{P_2}{16} \tag{63}$$

$$P_2 = \frac{30 \cdot 16 + 30 \cdot 4 + P_2}{16} \Longrightarrow 16P_2 = 480 + 120 + P_2 \tag{64}$$

$$P_2 = 40 \Longrightarrow P_1 = 40 \tag{65}$$

Then, we can plug these values into the each of the firms demand functions:

$$P_2 = 40 \implies P_1 = 40 \tag{66}$$

$$Q_1 = 100 - 2P_1 + P_2 \implies Q_1 = 100 - 2 \cdot 40 + 40 \tag{67}$$

$$Q_2 = 100 - 2P_2 + P_1 \implies Q_2 = 100 - 2 \cdot 40 + 40 \tag{68}$$

$$Q_1 = Q_2 = 60$$

Question 2.c

c. Suppose now Firm 1 sets its price first and then Firm 2 sets its price. What price will each firm charge, how much will it sell, and what will its profit be?

If Firm 1 sets its price first, it does so take Firm 2's reaction function into account. Firm 1's profit function is then:

$$\pi_1 = (P_1 - 10)Q_1 = (P_1 - 10)(100 - 2P_1 + (30 + \frac{P_1}{40})) \quad (69)$$

To find the profit maximising price, take the derivative of π_1 with respect to P_1, set it equal to 0, and solve for P_1:

$$\pi_1 = 100P_1 - 2P_1^2 + 30P1 + \frac{P_1^2}{40} - 1000 + 20P_1 - 300 - \frac{P_1}{4} \quad (70)$$

Solving for P_1:

$$P_1 = £42.14 \tag{71}$$

Then substitute P1 into Firm 2's reaction function to get P_2:

$$P_2 = 30 + \frac{P_1}{4} \implies P_2 = 30 + \frac{42.14}{4} \tag{72}$$

$$P_2 = £40.53 \tag{73}$$

At these prices, each firm's quantity is:

$$Q_1 = 100 - 84.28 + 40.53 = 56.25 \tag{74}$$

$$Q_2 = 100 - 81.06 + 42.14 = 61.08. \tag{75}$$

Profits, in turn, are:

$$\pi_1 = (42.14 - 10) \cdot 56.25 \longrightarrow 32.14 \cdot 56.25 = £1807.875 \tag{76}$$

$$\pi_2 = (40.53 - 10) \cdot 61.08 \Longrightarrow 30.53 \cdot 61.08 = £1864.772 \tag{77}$$

If Firm 1 sets its price first, both firms make greater profits than when they chose prices simultaneously.

Question 2.d

d. Suppose you are the owner of one of these firms and that there are three ways you could compete: (i) You and the other firms set price at the same time; (ii) You set price first; or (iii) Your competitor sets price first. If you could choose among these options, which one would you prefer? Explain why.

Answer: Your first choice should be (iii), and your second choice should be (ii). (Compare the profits in part (b), with those in part (c). From the reaction functions, we know that the price leader provokes a price increase in the follower. By being able to move second however, the follower undercuts the leader by raising price less than the leader. Note this is the opposite of what would happen in a Stackelberg-Cournot market.

Question 3.a

3.a. Consider two firms facing the demand curve P = 50-5Q, where $Q = Q_1 + Q_2$. The firms' cost functions are $C_1(Q_1) = 20 + 10Q_1$ and $C_2(Q_2) = 10 + 12Q_2$.

a. What is each firm's reaction curve, equilibrium output, and profit if they compete following the Cournot model.

Question 3.a

$$\pi_1(q_1, q_2) = p(Q)q_1 - c_1(q_1) \tag{78}$$

$$\pi_1(Q_1, Q_2) = PQ_1 - C_1 \tag{79}$$

$$P = 50 - 5Q \implies P = 50 - 5Q_1 - 5Q_2 \tag{80}$$

$$\pi_1(Q_1, Q_2) = PQ_1 - C_1 \implies (50 - 5Q_1 - 5Q_2)Q_1 - (20 + 10Q_1) \tag{81}$$

Deriving the profit function with respect to Q1 to zero, we find Firm 1's reaction function:

$$\pi_1(Q_1, Q_2) = (50 - 5Q_1 - 5Q_2)Q_1 - (20 + 10Q_1) \tag{82}$$

$$\pi_1(Q_1, Q_2) = 40Q_1 - 5Q_1^2 - 5Q_1Q_2 - 20. \tag{83}$$

$$\frac{\partial \pi_1(Q_1, Q_2)}{\partial Q_1} \implies Q_1 = 4 - \frac{Q_2}{2} \tag{84}$$

Similarly, Firm 2's profit function is:

$$\pi_2(Q_1, Q_2) = (50 - 5Q_1 - 5Q_2)Q_2 - (10 + 12Q_2) \tag{85}$$

$$\pi_2(Q_1, Q_2) = 38Q_2 - 5Q_2^2 - 5Q_1 Q_2 - 10. \tag{86}$$

$$\frac{\partial \pi_2}{\partial Q_2} = 38 - 10Q_2 - 5Q_1 = 0 \Longrightarrow Q_2 = 3.8 - \frac{Q_1}{2} \tag{87}$$

To find the Cournot equilibrium, substitute Firm 2's reaction function into Firm 1's reaction function (Next slide)

To find the Cournot equilibrium, substitute Firm 2's reaction function into Firm 1's reaction function (Next slide)

$$Q_1 = 4 - \frac{Q_2}{2} \tag{88}$$

$$Q_2 = 3.8 - \frac{Q_1}{2} \tag{89}$$

$$Q_1 = 4 - \frac{3.8 - \frac{Q_1}{2}}{2} \Longrightarrow 4 - \frac{7.6 + Q_1}{4} \tag{90}$$

$$Q_1 = 4 - \frac{7.6 + Q_1}{4} \Longrightarrow \frac{16 - 7.6 + Q_1}{4} \Longrightarrow \frac{8.4 + Q_1}{4} \tag{91}$$

$$Q_1 = \frac{8.4 + Q_1}{4} \tag{92}$$

$$4Q_1 = 8.4 + Q_1 \Longrightarrow 4Q_1 - Q_1 = 8.4 \tag{93}$$

$$3Q_1 = 8.4 \Longrightarrow Q_1 = 2.8 \tag{94}$$

Substituting this value for Q_1 into the reaction function for Firm 2, we find $Q_2 = 2.4$. Substituting the values for Q_1 and Q_2 into the demand function to determine the equilibrium price P = 50 - 5(2.8 + 2.4) = £24.

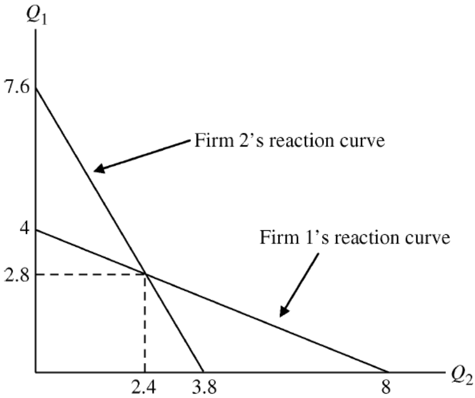
The profits for Firms 1 and 2 are equal to:

$$\pi_1 = 24 \cdot 2.8 - (20 + 10 \cdot 2.8) = £19.20 \tag{95}$$

$$\pi_2 = 24 \cdot 2.4 - (10 + 12 \cdot 2.4) = £18.80 \tag{96}$$

Total profits would then be $\pi_T = £38$.

The firms' reactio curves and the Cournot equilibrium are shown below:



$$Q_1 = 4 - \frac{Q_2}{2} \text{ and } Q_2 = 3.8 - \frac{Q_1}{2} \tag{97}$$

b. Suppose both firms have entered the industry already (so their fixed costs are sunk). If the firms collude to produce the joint profit-maximising level of output, how much will each firm produce? What will be the total profit of the cartel?

- If the firms collude, they face the market demand curve, so their marginal revenue curve is MR = 50 - 10Q
- The marginal costs of each firm are, respectively, $MC_1 = 10$ and $MC_2 = 12$
- The joint profit-maximising solution is for Firm 1 to produce all of the output because its marginal cost is less than Firm 2's

Setting marginal revenue equal to marginal cost (the marginal cost of Firm 1, since it is constant and lower than that of Firm 2) to determine the profit-maximising quantity, Q:

$$MR = MC_1 \tag{98}$$

$$50 - 10Q = 10 \implies Q = 4. \tag{99}$$

To obtain the price, replace the quantity in the inverse demand function:

$$P = 50 - 5(4) = 50 - 20 = £30 \tag{100}$$

The cartel would then obtain total profits:

$$\pi_T = 30 \cdot 4 - (20 + 10 \cdot 4) - (10 + 12 \cdot 0) = £50 \tag{101}$$

c. Suppose that under the cartel agreement, firms decide to share their joint profits equally. But when coming back home from the meeting with Firm 1, the owner of Firm 2 thinks that perhaps her firm will be better off by breaking the cartel agreement. What would be Firm 2's quantity and profit in that case assuming Firm 1 will stick to the agreement and produce its corresponding joint profit-maximising level of output you found in (b)?

- Under the cartel agreement Firm 2 receives £25 ($\frac{50}{2}$)
- Assuming that Firm 2 will stick to the agreement means that Firm 1 will produce 4 units.

.

The best response to that by Firm 2 is the one given by its best response:

$$Q_2 = 3.8 - \frac{Q_1}{2} \implies Q_2 = 3.8 - \frac{4}{2} \implies Q_2 = 1.8 \qquad (102)$$

The market price will then be:

$$P = 50 - 5(4 + 1.8) = £1 \tag{103}$$

The market price will then be P = 50 - 5(4 + 1.8) = £1.

The profits of Firm 2 will then be:

$$\pi_2 = 1 \cdot 1.8 - 10 + 12 \cdot 1.8 = £6.2. \tag{104}$$

Firm 2 will not break its collusive agreement with Firm 1.

d. How much should Firm 1 be willing to pay to purchase Firm 2 if collusion is illegal (or impossible) but a takeover is not?

Answer: To determine how much Firm 1 will be willing to pay to purchase Firm 2, we must compare Firm 1's profits in the monopoly situation versus it profits under oligopoly. The difference between the two will be what Firm 1 is willing to pay for Firm 2. From part (b), we know that if Firm 1's is the only producer and sets marginal revenue equal to its marginal cost, it will attain a profit of £60 (we do not need to subtract now Firm 2 fixed cost since now that firm no longer exists). This is what the firm would earn if it was a monopolist.

$$\pi_T = 30 \cdot 4 - (20 + 10 \cdot 4) - \cancelto{0}{(10 + 12 \cdot 0)} = £60 \qquad (105)$$

From part (a), we know that the profit of Firm 1 is £19.20 when firms compete against each other in the Cournot. Firm 1 should therefore be willing to pay up to £60 - £19.20 = £40.80 to take over Firm 2.

Microeconomics (5SSPP217)

Seminar Problem Set 9

Felipe Torres
felipe.torres@kcl.ac.uk

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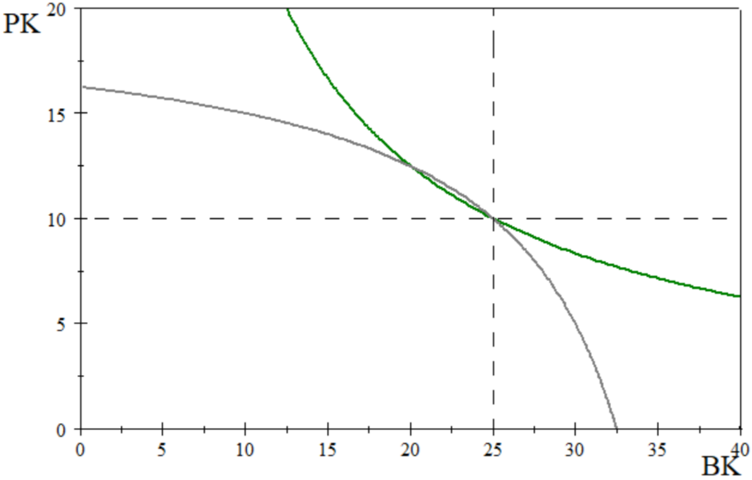
Question 1.a

1. Katherine and Gareth consume bananas and potatoes, where Bi and Pi denote their consumption of these goods respectively. Katherine's utility function is $U_K(B_K, P_K) = B_K P_K$ and Gareth's is $U_G(B_G, P_G) = (B_G)^{\frac{1}{2}}(P_G)^{\frac{1}{2}}$. Their initial endowments are 25 bananas and 10 potatoes for Katherine and 15 bananas and 10 potatoes for Gareth.

a. Draw an Edgeworth box with bananas in the horizontal axis and Katherine in the lower left corner. Illustrate the endowment point and the indifference curves for each agent which pass through the endowment poin t.


PK
20
15
10
5
0
0
5
10
15
20
25
30
35
40
BK

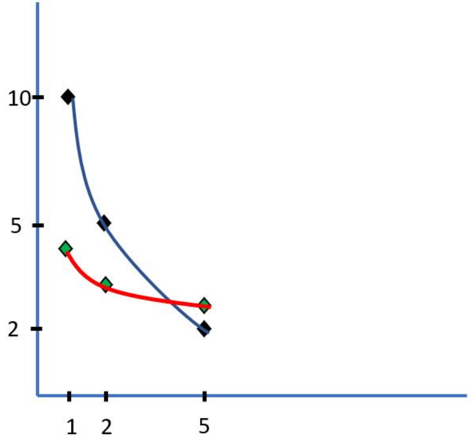
We have the following utility functions, in order to identify the shape of it, we can set a certain level of utility and change B_K and get P_K:

$$U_K(B_K, P_K) = B_K P_K \tag{1}$$

$$U_G(B_G, P_G) = (B_G)^{\frac{1}{2}} (P_G)^{\frac{1}{2}} \tag{2}$$

U(Bk Pk)	U(Bg,Pg)	B	Pk	Pg
10	10	5	2	2.11
10	10	2	5	2.67
10	10	1	10	3.16

Table: Utilities table


10
5
2
1
2
5

Question 1.b

b. What is Katherine's initial level of utility? What is Gareth's?

They tell us the original endowments:

$$U_K(B_K, P_K) = B_K P_K \tag{3}$$

$$U_K(B_K, P_K) = 25 \cdot 10 \implies U_K = 250 \tag{4}$$

$$U_G(B_G, P_G) = (B_G)^{\frac{1}{2}}(P_G)^{\frac{1}{2}} \tag{5}$$

$$U_G(B_G, P_G) = (15)^{\frac{1}{2}}(10)^{\frac{1}{2}} \implies U_G = 12.25 \tag{6}$$

c. Take the price of potatoes as a numeraire, i.e. $P_p = 1$. Find Gareth's and Katherine's demands for bananas and potatoes as a function of the prices of the price of bananas p_B.

Gareth and Katherine have straightforward Cobb-Douglas utilities. Katherine's budget is $25p_B + 10$ whereas Gareth's budget is $15p_B + 10$. Using what you know about consumer theory from Lectures 1-3 you should be able to get the demands:

$$B_K^* = \frac{25p_B + 10}{2p_B}, P_K^* = \frac{25p_B + 10}{2} \tag{7}$$

$$B_G^* = \frac{15p_B + 10}{2p_B}, B_G^* = \frac{15p_B + 10}{2} \tag{8}$$

For a Cobb-Douglas utility function is (lecture 2, slide 55)

$$X_1^* = \frac{am}{(a+b)p1} \text{ where } U(x_1, x_2) = x_1^a x_2^b \text{ and } m = p_1 x_1 + p_2 x_2 \tag{9}$$

Question 1.d

d. What is the equilibrium price of bananas? What is the allocation of bananas and potatoes which will result?

In equilibrium it must be that:

$$B_K^* + B_G^* = \frac{25p_B + 10}{2p_B} + \frac{15p_B + 10}{2p_B} = 40 \tag{10}$$

where 40 is the total endowment: $25 + 15 = 40$

$$P_G^* + P_K^* = \frac{15p_B + 10}{2} + \frac{25p_B + 10}{2} = 20 \tag{11}$$

where 40 is the total endowment: $10 + 10 = 20$

The equilibrium price of bananas is the one that makes any of the above equations hold. By Walras' Law, if one market is in equilibrium the other must be too.

Let's solve for p_B

$$B_K^* + B_G^* = \frac{25p_B + 10}{2p_B} + \frac{15p_B + 10}{2p_B} = 40 \tag{12}$$

$$B_K^* + B_G^* = \frac{25p_B + 10 + 15p_B + 10}{2p_B} = 40 \tag{13}$$

$$B_K^* + B_G^* = \frac{40p_B + 20}{2p_B} = 40 \tag{14}$$

$$B_K^* + B_G^* = 40p_B + 20 = 80p_B \Longrightarrow 20 = 80p_B - 40p_B \tag{15}$$

$$B_K^* + B_G^* = 40p_B = 20 \Longrightarrow p_B = \frac{1}{2} \tag{16}$$

This results in the equilibrium allocation given $p_B = \frac{1}{2}$

$$B_K^* = \frac{25p_B + 10}{2p_B}, P_K^* = \frac{25p_B + 10}{p_B} \tag{17}$$

$$B_K^* = \frac{25\frac{1}{2} + 10}{2\frac{1}{2}}, P_K^* = \frac{25\frac{1}{2} + 10}{2} \tag{18}$$

$$B_K^* = \frac{12.5 + 10}{1}, P_K^* = \frac{12.5 + 10}{2} \tag{19}$$

$$B_K^* = 22.5, P_K^* = 11.25 \tag{20}$$

You can try to solve B_G and P_G

$$B_K^* + B_G^* = 40 \Longrightarrow 22.5 + B_G^* = 40 \Longrightarrow B_G^* = 17.5 \tag{21}$$

$$P_K^* + P_G^* = 20 \Longrightarrow 11.25 + P_G^* = 20 \Longrightarrow P_G^* = 8.75 \tag{22}$$

Question 1.e

e. What are their levels of utility after trading?

Question 1.e

$$U_K(B_K, P_K) = B_K P_K \tag{23}$$

$$U_K(B_K, P_K) = 22.5 \cdot 11.25 \implies U_K = 253.13 \tag{24}$$

$$U_G(B_G, P_G) = B_G^{\frac{1}{2}} P_G^{\frac{1}{2}} \tag{25}$$

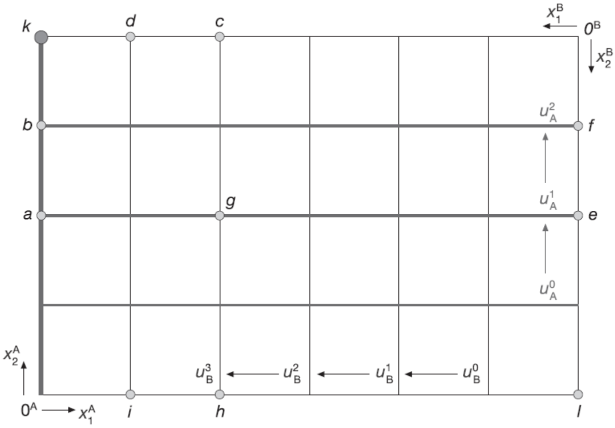
$$U_G(B_G, P_G) = (17.5)^{\frac{1}{2}}(8.75)^{\frac{1}{2}} \implies U_G = 12.37 \tag{26}$$

Both are better off than with their initial endowments. $U_K = 250$ and $U_G = 12.25$

Question 2.a

2. Consider a pure exchange economy with two individuals (A and B) and two goods (1 and 2). Suppose that individual A only derives utility from good 2 and good 1 is completely neutral to this individual. On the other hand, consumer B only derives utility from good 1 and does not care at all about good 2.

a. Using an Edgeworth box, find the set of Pareto efficient allocations in this economy.


k
d
c
x_1^B
0^B
x_2^B
u_A^2
b
f
g
u_A^1
a
e
u_A^0
x_2^A
u_B^3
u_B^2
u_B^1
u_B^0
0^A
x_1^A
i
h
l

- It is clear that no interior allocation such as g can be Pareto efficient because any allocation that gives A more of good 2 (upward) and B more of good 1 (leftward) leaves both consumers better off.
- By the same token, the allocations on the sides of the box are not Pareto efficient either because more of good 2 can be given to A without hurting B (like when moving from a to b or from e to f) or because more of good 1 can be given to B without hurting A (like when moving from c to d or from h to l).
- Following this reasoning, we can conclude that the only Pareto efficient allocation in this economy is point k, where consumer A has all good 2 and consumer B all good 1

b. Consider now that goods can be traded in the market at prices p_1 and p_2. Assume that person A owns all E_1 units of good 1 there are in the economy whilst person B owns all E_2 units of good 2 there are in the economy. Find the demands of these consumers for both goods. Find the equilibrium ratio of prices p1/p2 of this economy. Is the resulting equilibrium allocation Pareto efficient?

For consumer A:

- Because consumer A only cares about good 2, she is not going to demand any good 1, $x_1 A* = 0$
- A is going to spend their entire budget in good 2
- The budget of person A is $E_1 p_1$
- Consumer A demands $x_2^{A*} = \frac{E_1 p_1}{p_2}$

By the same token:

- Consumer B demands nothing of good 2 and spends their entire income E2p2 in good 1 $x_2^{B*} = 0$
- Consumer B demands $x_1^{B*} = \frac{E_2 p_2}{p_1}$

Note that this is indeed the Pareto efficient allocation. The prices that clear the market must be such that

$$x_1^{A*} + x_1^{B*} = 0 + \frac{E_2 p_2}{p_1} = E_1 \longrightarrow \frac{p_2}{p_1} = \frac{E_1}{E_2} \tag{27}$$

$$x_2^{A*} + x_2^{B*} = \frac{E_2 p_2}{p_1} + 0 = E_2 \frac{p_2}{p_1} = \frac{E_1}{E_2} \tag{28}$$

This means that in equilibrium $\frac{p_1}{p_2} = \frac{E_1}{E_2}$

This is the slope of the budget line which must pass through the initial endowment point (point I in the graph) and also through the only Pareto efficient allocation (point k). Given that budget line, consumers indeed demand the Pareto efficient allocation. The equilibrium is indeed efficient.

3. Edgeworth (E) and Bowley (B) are stranded in a desert island. Edgeworth has a bag with four apples (A) and Bowley owns two coconuts (C). They decide to start a market and trade these goods. Assume that coconuts are the numeraire good, i.e. $p_C = 1$. Edgeworth's and Bowley's preferences are given by the following utility functions:

$$U_B = \frac{1}{2} \ln C_B + \frac{1}{2} \ln A_B \tag{29}$$

$$U_E = \frac{1}{3} \ln C_B + \frac{2}{3} \ln A_E \tag{30}$$

a. Find Bowley's and Edgeworth's demands for coconuts and apples.

- Let's call p the price of apples
- Edgeworth's budget: 4 apples $\times p$
- Bowley's budget: $2 \cdot 1 = 2$

Using what you know about consumer theory you should be able to get demands[1]:

$$A_B = \frac{1}{p}, C_B = 1 \tag{31}$$

$$A_E = \frac{8}{3}, C_E = \frac{4p}{3} \tag{32}$$

[1] See the appendix where I derived the demand functions from the utility functions

But from lecture 2, slides 37-44

$$\frac{\frac{\partial U}{\partial x_1}}{\frac{\partial U}{\partial x_2}} = MRS = \frac{p_1}{p_2} \tag{33}$$

But let's go back to lecture 2, slides 37-43:

$$\frac{\partial U}{\partial A_B} = \frac{1}{2A_B} \tag{34}$$

$$\frac{\partial U}{\partial C_B} = \frac{1}{2C_B} \tag{35}$$

$$MRS = \frac{\frac{1}{2A_B}}{\frac{1}{2C_B}} = \frac{p}{1} \Longrightarrow \frac{2C_B}{2A_B} = p \tag{36}$$

$$MRS = C_B = pA_B \tag{37}$$

We know that the budget subject B:

$$2 \cdot 1 = A_B p + C_B \cdot 1 \tag{38}$$

$$2 = A_B p + pA_B \implies 2A_B p = 2 \implies A_B = \frac{1}{p} \tag{39}$$

Now, for C_B:

$$2 = p\frac{1}{p} + C_B \implies 2 = \cancelto{1}{p}\frac{1}{\cancelto{1}{p}} + C_B \tag{40}$$

$$C_B = 1 \tag{41}$$

You can do the same for Edgeworth.

Question 3.b

b. Find the competitive equilibrium of this exchange economy and the resulting equilibrium allocation.

Therefore, the equilibrium price of apples must be such that:

$$A_B + A_E = \frac{1}{p} + \frac{8}{3} = 4 \tag{42}$$

$$C_B + C_E = 1 + \frac{4p}{3} = 2 \tag{43}$$

Solving for one or the other equation you should get $p^* = \frac{3}{4}$

$$A_B + A_E = \frac{1}{p} + \frac{8}{3} = 4 \tag{44}$$

$$A_B + A_E = \frac{1}{p} = 4 - \frac{8}{3} \implies \frac{1}{p} = \frac{12 - 8}{3} \implies \frac{1}{p} = \frac{4}{3} \tag{45}$$

$$p^* = \frac{3}{4} \tag{46}$$

The resulting equilibrium allocation using $p^* = \frac{3}{4}$ is:

$$A_B^* = \frac{1}{\frac{3}{4}}, C_B^* = 1 \Longrightarrow A_B^* = \frac{4}{3}, C_B^* = 1 \tag{47}$$

$$A_E^* = \frac{8}{3}, C_E^* = \frac{4\frac{3}{4}}{3} \Longrightarrow A_E^* = \frac{8}{3}, C_E^* = 1 \tag{48}$$

Question 3.c

c. Find the expression of the contract curve for this economy and use your answer to check that the equilibrium allocation you found in (b) is indeed Pareto optimal.

Because the utility functions are well behaved (convex preferences, strongly monotonic) we know that Pareto efficiency will take place where both MRS coincide.

Let's obtain the MRS:

$$U_B = \frac{1}{2} \ln C_B + \frac{1}{2} \ln A_B \tag{49}$$

$$MU_{BC} = \frac{\partial U_B}{\partial C_B} = \frac{1}{2} \cdot \frac{1}{C_B} \tag{50}$$

$$MU_{BA} = \frac{\partial U_B}{\partial A_B} = \frac{1}{2} \cdot \frac{1}{A_B} \tag{51}$$

$$MRS_B = \frac{MU_{BC}}{MU_{BA}} = \frac{A_B}{C_B} \tag{52}$$

Let's obtain the MRS for E:

$$U_E = \frac{1}{3} \ln C_B + \frac{2}{3} \ln A_E \tag{53}$$

$$MU_{EC} = \frac{\partial U_E}{\partial C_E} = \frac{1}{3} \cdot \frac{1}{C_E} \tag{54}$$

$$MU_{EA} = \frac{\partial U_E}{\partial A_E} = \frac{2}{3} \cdot \frac{1}{A_E} \tag{55}$$

$$MRS_E = \frac{MU_{EC}}{MU_{EA}} = \frac{A_E}{2C_E} \tag{56}$$

For consumer A their MRS is $\frac{A_B}{C_B}$ whereas for consumer B their MRS is $\frac{A_E}{2C_E}$.

Hence, the expression for the contract curve is $\frac{A_B}{C_B} = \frac{A_E}{2C_E}$.

It is easy to see that the equilibrium allocation found in (a) is Pareto efficient as $\frac{A_B^*}{C_B^*} = \frac{4}{3} = \frac{A_E^*}{2C_E^*}$.

$$\frac{\frac{4}{3}}{1} = \frac{\frac{8}{3}}{2 \cdot 1} = \frac{4}{3} \tag{57}$$

Question 3.d

d. Find the transfer T of apples from Edgeworth to Bowley which supports as an equilibrium of this economy the Pareto efficient allocation where both individuals consume two apples each

First we need to have clear which allocation we have been asked to support as an equilibrium. Using the equation of the contract curve, we can establish that at the Pareto efficient allocation where both individuals consume two apples each, their consumption of coconuts must be such that:

$$\frac{2}{C_B} = \frac{2}{2C_E} \Longrightarrow \frac{2}{C_B} = \frac{1}{2 - C_B} \Longrightarrow C_B = \frac{4}{3} \Longrightarrow C_E = \frac{2}{3} \tag{58}$$

$$A_B = A_E = 2 \tag{59}$$

$$C_B = \frac{4}{3}, C_E = \frac{2}{3} \tag{60}$$

Next, the budget of Edgeworth after the transfer:

- Edgeworth's budget: (4-T)p
- Bowley's budget: 2+Tp

We want to find the transfer T and the equilibrium price p for which our consumers demand exactly the allocation we want to support:

$$A_B = A_E = 2 \tag{61}$$

$$C_B = \frac{4}{3}, C_E = \frac{2}{3} \tag{62}$$

Again, using the techniques we saw in consumer theory you should be able to show that the demands of Edgeworth and Bowley under their new budgets are:

$$A_B = \frac{1}{p} + \frac{T}{2}, C_B = 1 + \frac{Tp}{2} \tag{63}$$

$$A_E = \frac{8 - 2T}{3}, C_E = \frac{(4 - T)p}{3} \tag{64}$$

How do get these demand functions?

$$MU_{AE} = \frac{\partial U_E}{\partial A_E} = \frac{2}{3} \cdot \frac{1}{2A_E} \tag{65}$$

$$MU_{CE} = \frac{\partial U_E}{\partial C_E} = \frac{1}{3} \cdot \frac{1}{C_E} \tag{66}$$

How do get these demand functions?

$$MRS_E = \frac{\frac{2}{3A_E}}{\frac{1}{3C_E}} = \frac{p}{1} \Longrightarrow C_E = \frac{A_E p}{2} \tag{67}$$

Edgeworth's new budget function is:

$$(4-T)p = A_E p + \frac{A_E p}{2} \cdot 1 \Longrightarrow (4-T)p = \frac{2A_E p + A_E p}{2} \tag{68}$$

$$3A_E p = 2(4-T)p \Longrightarrow A_E = \frac{8-2T}{3} \tag{69}$$

Again, using the techniques we saw in consumer theory you should be able to show that the demands of Edgeworth and Bowley under their new budgets are:

$$A_B = \frac{1}{p} + \frac{T}{2}, C_B = 1 + \frac{Tp}{2} \tag{70}$$

$$A_E = \frac{8-2T}{3}, C_E = \frac{(4-T)p}{3} \tag{71}$$

Using $A_E = 2$, we get that $T^* = 1$.

$$2 = \frac{8-2T}{3} \implies 6 = 8-2T \implies 2T = 2 \tag{72}$$

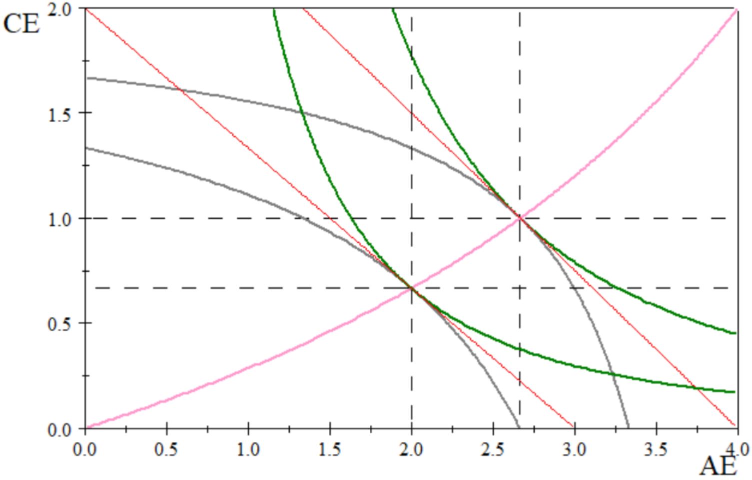
$$T^* = 1 \tag{73}$$

For completeness, we check new equilibrium price is $p^* = \frac{2}{3}$ and that with that transfer and price of apples, our consumer demand exactly the allocation we want them to demand.

You need to plug in the this price into the demand functions and see that markets clear.

Question 3.e

e. Draw this economy in an Edgeworth box with apples in the horizontal axis and Edgeworth in the lower left corner. Include the equilibrium allocation you found in (b), the contract curve you found in (c) and the new endowment allocation and equilibrium you found in (d).


CE
2.0
1.5
1.0
0.5
0.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
AE

The pink curve is the contract curve. To draw it you should have used the fact that:

$$A_B + A_E = 4 \tag{74}$$

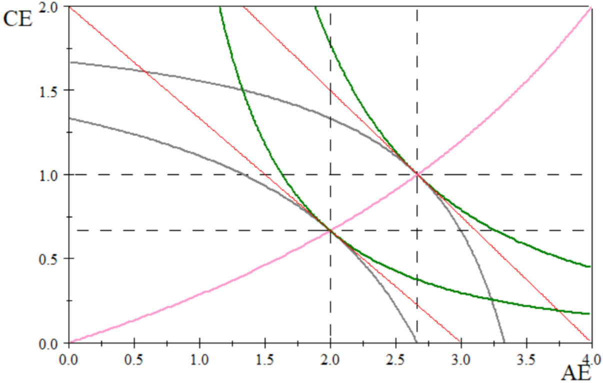
$$C_B + C_E = 2 \tag{75}$$

in the equation of the contract curve we found above to obtain:

$$A_B + A_E = 4 \text{ and } C_B + C_E = 2 \tag{76}$$

$$A_B = 4 - A_E \text{ and } C_B = 2 - C_E \tag{77}$$

$$\frac{A_B}{C_B} = \frac{(4 - A_E)}{(2 - C_E)} = \frac{A_E}{2C_E} \Longrightarrow C_E = \frac{2A_E}{(8 - A_E)} \tag{78}$$



The red line on the right is the budget line corresponding to the equilibrium without the transfer. The red line on the left is the budget line corresponding to the equilibrium with the transfer. The initial endowment allocation is the one on the lower right corner, whereas the one after the transfer is at (3,0) (or (1,2) if you read the graph from Bowley's corner).

Question 4.a

4. Anil and Becca are the only people living in a village. They consume only two goods, apples (a) and bananas (b). Anil has an initial endowment of 10 apples and 30 bananas. Becca has an initial endowment of 50 apples and 50 bananas. Anil's utility function is $U^A(x_a^A, x_b^A) = \min\{x_a^A, x_b^A\}$ and Becca's utility function is $u^B(x_a^B, x_b^B) = \min\{x_a^B, x_b^B\}$. Would the initial allocation be Pareto efficient? What is the set of Pareto efficient allocations of this economy?

Question 4.a

Let's see the level of utility they get with their initial endowment:

$$U_A(10, 30) = 10 \text{ and } U_B(50, 50) = 50 \tag{79}$$

If we transferred N apples from Anil to Becca, we would not increase Becca's utility and we would decrease Anil's:

$$U_A \min(10 - N, 30) = 10 - N \text{ and } U_B \min(50 + N, 50) = 50 \quad (80)$$

If we transferred bananas from Anil to Becca, we would not change their utilities (or we would decrease Anil's utility if we transferred enough bananas):

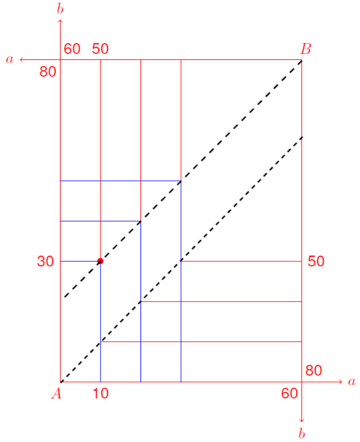
$$U_A(10, 30 - N) = 10 \text{ and } U_B(50, 50 + N) = 50 \text{ for N} \leq 20 \quad (81)$$

If we transferred bananas from Becca to Anil, we would not change Anil's utility and just decrease Becca's:

$$U_A(10, 30 + N) = 10 \text{ and } U_B(50, 50 - N) = 50 - N \text{ for N } \geq 1 \quad (82)$$

Then, starting from the initial allocation, it is impossible to find an alternative allocation that would increase the utility of one of the two agents without reducing the utility of the other. The initial allocation is therefore Pareto optimal.

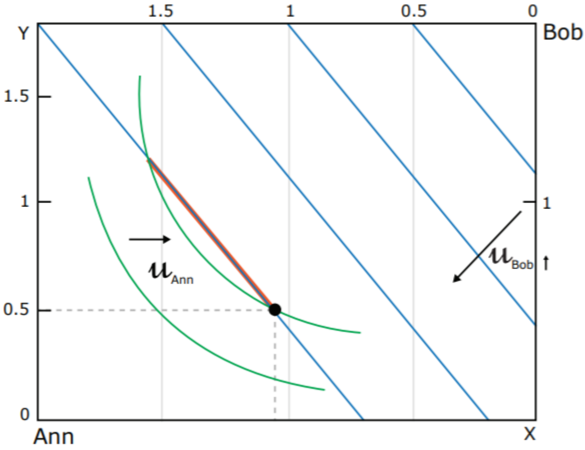
We can use this reasoning to conclude that all allocations between the two rays collecting the 2kinks" of Anil's and Becca's indifference curves (the two dashed lines in the Edgeworth box below) is the set of Pareto efficient allocations. Moving away from any of those points increases the utility of one individual only to hurt the welfare of the other individual or does not change utilities at all (when we move vertically along two overlapping indifference curves). Hence, the contract "curve" is an area in this case.



Question 5.a

5. Consider an economy with two goods - X and Y - and two agents – Ann and Bob. Suppose Ann is endowed with one unit of X and half a unit of Y, i.e. $w^A = (1, 0.5)$ and Bob is endowed with one unit of X and one and a half units of Y, i.e. $wB = (1, 1.5)$. Additionally, suppose their utility functions are given by $U_A(X_A, Y_A) = X_A Y_A$ and $U_B(X_B, Y_B) = Y_B + 2X_B$.

a. Draw an Edgeworth box with good X in the horizontal axis and Ann in the lower left corner. Illustrate the endowment point and the indifference curves for each agent which pass through the endowment point.


1.5
1
0.5
0
Y
Bob
1.5
1
0.5
0
Ann
X
1
u_{Ann}
u_{Bob}

b. Find the set of Pareto efficient allocations in this economy and depict these in the Edgeworth box.

The contract curve is the set of allocations such that by moving away from them, at least one of the individuals is made worse off. Visually, this is given by the set of tangencies of the consumers' indifference curves. At these tangencies, the consumers have the same MRS between X and Y.

Thus, let's get the MRSs:

$$MU_{AX} = Y_A \text{ and } MU_{AY} = X_A \Longrightarrow MRS = \frac{Y_A}{X_A} \tag{83}$$

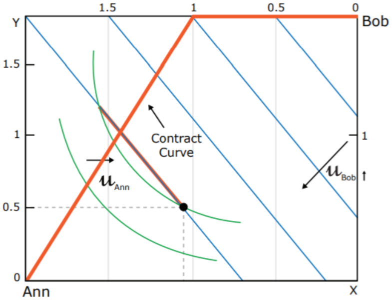
$$MU_{BX} = 2 \text{ and } MU_{BY} = 1 \Longrightarrow MRS = \frac{2}{1} \tag{84}$$

Hence, the tangency occurs at all allocations where[2]:

$$MRS_A = MRS_B \Longrightarrow \frac{Y_A}{X_A} = 2 \Longrightarrow Y_A = 2X_A \tag{85}$$

[2](represented by positively sloped orange line in the graph below).

Note, however, that in this economy, there are only 2 units of good X and 2 units of good Y, therefore, this equation cannot hold when $X_A > 1$. For $X_A > 1$, the contract curve is just the top edge of the box (represented by the flat orange segment in the graph below)).



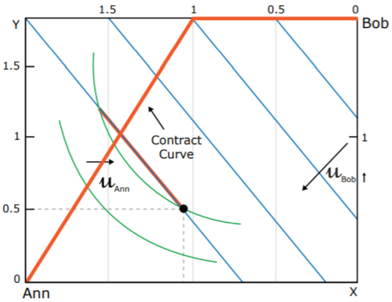
Why? At those points where $Y_A = 2$ and $X_A > 1$ the MRS for Ann is lower than for Bob. In other words, Anne is relatively more interested in good Y (relatively high MU of good Y drives the low MRS) than Bob (who has a relatively high MU of good X which drives his MRS up).[3]

Remember that MRS is equal to:

$$MRS = \frac{MU_x}{MU_y} \tag{86}$$

[3] A MRS of 2, means that Bob is willing to trade two units of Y for one unit of X. This means that he values more X over Y. The general statement is as follows: $MRS = \frac{\Delta Y}{\Delta X}$ is the marginal rate of substitution of good X for good Y.

They would benefit from exchanging good X and Y; Ann would gladly accept more good Y in exchange for good X and Bob would gladly accept some more good X in exchange for some good Y. The problem is that Anne already has all good Y and Bob has none left. Moving away from any point at the top edge of the box with $X_A > 1$ would leave Anne, Bob or both of them worse off. Hence, all those points are also Pareto efficient and are part of the contract curve.



Question 5.c

c. Find the demands for Ann and Bob as a function of the prices p_x and p_y.

Anne has standard Cobb-Douglas utility so her maximisation problem should be straightforward. You should be able to show that her consumption demands are:

$$X_A^* = \frac{p_x + 0.5p_y}{2p_x}, Y_A^* = \frac{p_x + 0.5p_y}{2p_y} \tag{87}$$

Bob has perfect substitutes preferences so we know that his demand will depend on the relationship between his MRS (2) and the ratio of prices.

- if $2 > \frac{p_x}{p_y}$ then, Bob will demand no good of Y and will only demand X (Good Y is relatively more expensive)
- if $2 < \frac{p_x}{p_y}$ then, Bob will demand no good of X and will only demand Y (Good X is relatively more expensive)

If $2 > \frac{p_x}{p_y}$:

$$X_B^* = \frac{p_x + 1.5p_y}{p_x}, Y_B^* = 0 \tag{88}$$

If $2 < \frac{p_x}{p_y}$:

$$X_B^* = 0, Y_B^* = \frac{p_x + 1.5p_y}{p_y} \tag{89}$$

If $2 = \frac{p_x}{p_y}$, then Bob is indifferent amongst all his just affordable consumption bundles.

Question 5.d

d. Find the equilibrium price ratio of this economy and the corresponding equilibrium allocation. Is it Pareto efficient?

Question 5.d

Suppose that the prices of X and Y are such that $2 > \frac{p_x}{p_y}$. Then, for the markets to clear we would need:

$$X_A^* + X_B^* = \frac{p_x + 0.5p_y}{2p_x} + \frac{p_x + 1.5p_y}{p_x} = 2 \tag{90}$$

We need to solve for $\frac{p_x}{p_y}$:

$$\frac{p_x + 0.5p_y + 2p_x + 3p_y}{2p_x} = 2 \implies 3p_x + 3.5p_y = 4p_x \tag{91}$$

$$3.5p_y = 4p_x - 3p_x \implies 3.5p_y = p_x \implies \frac{p_x}{p_y} = 3.5 \tag{92}$$

Solving for any of these two yields $\frac{p_x}{p_y} = 3.5$, which contradicts $2 > \frac{p_x}{p_y}$. Hence, the equilibrium is not here.

Suppose alternatively that $2 < \frac{p_x}{p_y}$. Then, for the markets to clear we would need:

$$X_A^* + X_B^* = \frac{p_x + 0.5p_y}{2p_x} + 0 = 2 \tag{93}$$

$$Y_A^* + Y_B^* = \frac{p_x + 0.5p_y}{2p_y} + \frac{p_x + 1.5p_y}{2p_y} = 2 \tag{94}$$

We need to solve for $\frac{p_x}{p_y}$:

$$\frac{p_x + 0.5p_y}{2p_x} + 0 = 2 \Longrightarrow p_x + \frac{1p_y}{2} = 4p_x \tag{95}$$

Solving for any of these two yields $\frac{p_x}{p_y} = \frac{1}{6}$, which contradicts $2 > \frac{p_x}{p_y}$. Hence, the equilibrium is not here either.

The only alternative left is $2 = \frac{p_x}{p_y}$. With that, Alice demands:

$$X_A^* = \frac{p_x + 0.5p_y}{2p_x} \tag{96}$$

$$X_A^* = \frac{p_x}{2p_x} + \frac{0.5p_y}{2p_x} \implies \frac{1}{2} + \frac{0.5}{2} \cdot \frac{p_y}{p_x} \tag{97}$$

$$X_A^* = \frac{1}{2} + \frac{1}{4} \cdot \frac{1}{2} \implies \frac{1}{2} + \frac{1}{8} \implies X_A^* = \frac{4+1}{8} \tag{98}$$

$$X_A^* = \frac{5}{8} \tag{99}$$

The only alternative left is $2 = \frac{p_x}{p_y}$. With that, Alice demands:

$$Y_A^* = \frac{p_x + 0.5p_y}{2p_y} \tag{100}$$

$$Y_A^* = \frac{p_x}{2p_y} + \frac{0.5p_y}{2p_y} \tag{101}$$

$$Y_A^* = \frac{1}{2}\frac{p_x}{p_y} + \frac{0.5}{2}\frac{p_y}{p_y} \implies \frac{1}{2}\cdot 2 + \frac{1}{4}\cdot 1 \tag{102}$$

$$Y_A^* = 1 + \frac{1}{4} \tag{103}$$

$$Y_A^* = \frac{5}{4} \tag{104}$$

Bob is indifferent between all his just affordable bundles including:

$$X_B^* = 2 - X_A^* \tag{105}$$

$$X_B^* = 2 - \frac{5}{8} \Longrightarrow X_B^* = \frac{11}{8} \tag{106}$$

$$Y_B^* = 2 - Y_A^* \tag{107}$$

$$Y_B^* = 2 - \frac{5}{4} \Longrightarrow Y_B^* = \frac{3}{4} \tag{108}$$

Which clear all markets. Observe that this allocation is Pareto efficient since it is on the contract curve:

$$Y_A^* = \frac{5}{4} = 2X_A^* \text{ and } (X_A^* < 1). \tag{109}$$

Microeconomics (5SSPP217)

Seminar 10

Felipe Torres
felipe.torres@kcl.ac.uk

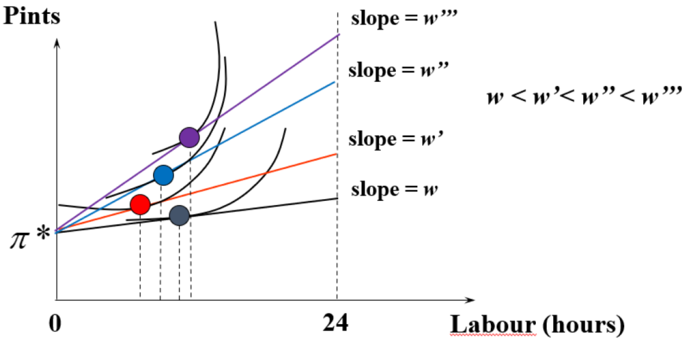
June 4, 2025

Question 1

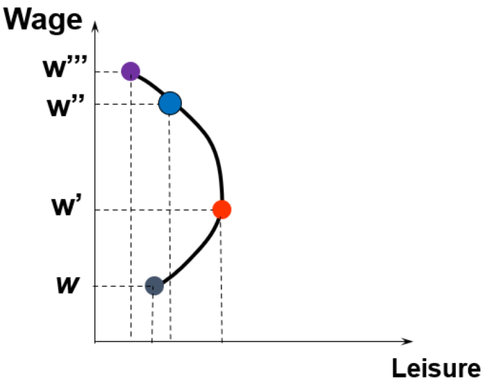
1. In a series of speeches in the late 1920s, John Maynard Keynes forecasted that, thanks to technological progress and the resulting increases in productivity, people would work only 15 hours a week by 2030. Assuming leisure and consumption are normal goods, use the graphical apparatus we developed for Mr Hyde's consumption-leisure problem in Lecture 19 and the concepts of income and substitution effect we saw in Lecture 3 to study what would happen to Hyde's labour supply if the wage w increased (due to a productivity improvement). What Keynes got wrong according to this model? Could Hyde's working hours even increase because of technological progress?

- An increase in the wage creates a substitution and an income effect.
- The opportunity cost of leisure increases; by not working, Hyde now foregoes more money per hour than before.
- Hence, Hyde will tend to demand less leisure, i.e. work more, and consume more beer
- But there is also an income effect at play. The increase in wage means that an hour of labour pays more.
- So, for instance, Hyde can buy the same beer he was buying before by working less. That positive income effect leads Hyde to consume more beer and to demand more leisure, i.e. work less.

- In other words, for work/leisure, the income and substitution effects go in opposite directions.
- What Keynes got wrong is that he expected the income effect to be much stronger than the substitution effect (red point, in the graph that we will see shortly).
- This might have been the case during the 20th century. But by the 1990s that was not true anymore, probably due to the fact that the marginal utility of leisure had become very low by then (blue point).
- If the substitution effect outweighs the income effect, working hours may even increase as the wage grows (purple point) and the demand curve for leisure may bend backwards (second graph).



- What Keynes got wrong is that he expected the income effect to be much stronger than the substitution effect (red point)
- This might have been the case during the 20th century. But by the 1990s that was not true anymore, probably due to the fact that the marginal utility of leisure had become very low by then (blue point).



- If the substitution effect outweighs the income effect, working hours may even increase as the wage grows (purple point) and the demand curve for leisure may bend backwards (second graph).

Question 1.a

1. Consider an economy with one consumer, called Clark, and one firm. Clark likes Beer (B) and leisure. He has 3 units of time. Hence, leisure is just $3 - L_c$, where L_c is the amount of labour Clark decides to sell to the firm. Clark is also the owner of the firm (so receives all its profits). His preferences are given by the following utility function

$$U = \frac{1}{2}\ln(3 - L_c) + \frac{1}{2}\ln(B_c) \qquad (1)$$

Question 1.a

a. The firm produces Beer out of the labour it demands from Clark (L_f) following the production technology $B_f = 2\sqrt{L_f}$. Assume the wage is equal to 1 and denote the price of Beer by p.

a. Write Clark´s budget constraint.

We know that this type of economy, where there is only one consumer and one producer (add slide) Clark's budget constraint will be:

$$pB_c = \pi + L_c \times 1 \tag{2}$$

$$pB_c = \pi + L_c \tag{3}$$

Question 2.b

b. Find Clark´s supply of labour

In order to find Clark's supply of labour, we need to solve the Clark's utility maximisation problem:

$$\max U(B_c, L_c) = \frac{1}{2}\ln(3 - L_c) + \frac{1}{2}\ln(B_c) \tag{4}$$

$$\text{s.t } pB_c = \pi + L_c \times 1 \tag{5}$$

Using the Lagrange method or by maximising the above utility function with respect to L_c after substituting by $B_c = \frac{\pi + L_c}{p}$ you should be able to find that Clark's demands for beer and supply of labour.

Let's solve this maximisation problem using the Lagrangian method:

$$\mathcal{L} = \frac{1}{2}\ln(3 - L_c) + \frac{1}{2}\ln(B_c) + \lambda(pB_c - \pi - L_c) \tag{6}$$

$$\frac{\partial \mathcal{L}}{\partial L_c} = \frac{1}{3 - L_c} \cdot -1 + \lambda(-L_c) = 0 \tag{7}$$

$$\frac{\partial \mathcal{L}}{\partial L_c} = -\frac{1}{3 - L_c} - \lambda = 0 \tag{8}$$

$$\lambda = -\frac{1}{3 - L_c} \tag{9}$$

$$\mathcal{L} = \frac{1}{2}\ln(3 - L_c) + \frac{1}{2}\ln(B_c) + \lambda(pB_c - \pi - L_c) \tag{10}$$

$$\frac{\partial \mathcal{L}}{\partial B_c} = \frac{1}{B_c} + \lambda p = 0 \tag{11}$$

$$\lambda = -\frac{1}{pB_c} \tag{12}$$

$$\lambda = -\frac{1}{3 - L_c} \tag{13}$$

$$\lambda = -\frac{1}{pB_c} \tag{14}$$

$$-\frac{1}{3 - L_c} = -\frac{1}{pB_c} \Longrightarrow B_c = \frac{3 - L_c}{p} \tag{15}$$

We plug B_c into the third order condition:

$$pB_c - \pi - L_c = 0 \tag{16}$$

$$B_c = \frac{3 - L_c}{p} \tag{17}$$

We plug B_c into the third order condition:

$$pB_c - \pi - L_c = 0 \tag{18}$$

$$\cancel{p}^{1}\left(\frac{3 - L_c}{\cancel{p}^{1}}\right) - \pi - L_c = 0 \tag{19}$$

$$3 - L_c - \pi - L_c \Longrightarrow 3 - \pi = 2L_c \Longrightarrow \frac{3 - \pi}{2} = L_c \tag{20}$$

$$L_c^* = \frac{3 - \pi}{2} \text{ if you solve for } B_c^* = \frac{3 + \pi}{2p} \tag{21}$$

Question 1.c

c. Find the firm´s demand for labour and its resulting profit

The firm demands labour L_f to maximise its profit:

$$\max \pi = p \cdot 2\sqrt{L_f} - L_f \times 1 \tag{22}$$

$$\frac{\partial \pi}{\partial L_f} = 2p\frac{1}{2}L_f^{\frac{1}{2}-1} - 1 \Longrightarrow 2p\frac{1}{2}\frac{1}{\sqrt{L_f}} - 1 \Longrightarrow p\frac{1}{\sqrt{L_f}} - 1 = 0 \tag{23}$$

$$p\frac{1}{\sqrt{L_f}} - 1 = 0 \Longrightarrow p\frac{1}{\sqrt{L_f}} = 1 \Longrightarrow p = \sqrt{L_f} \tag{24}$$

$$\sqrt{L_f} = p \tag{25}$$

$$L_f = p^2 \tag{26}$$

and output supply is equal to:

$$L_f = p^2 \tag{27}$$

$$B_f = 2\sqrt{L_f} \implies B_f = 2\sqrt{p^2} \tag{28}$$

$$B_f^* = 2p \tag{29}$$

Our profit function is:

$$\pi = p \cdot 2p - p^2 \implies 2p^2 - p^2 \tag{30}$$

$$\pi^* = p^2 \tag{31}$$

Question 2.d

d. Find the price of Beer p that clears markets.

The equilibrium price of beer is the one that clears all markets, that is, the price p^* such that (where $pi = p^2$

$$L_c^* = \frac{3 - \pi}{2} \text{ and } B_c^* = \frac{3 + \pi}{2p} \tag{32}$$

$$L_c^* = \frac{3 - p^2}{2} \text{ and } B_c^* = \frac{3 + p^2}{2p} \tag{33}$$

Must equal to the supply:

$$L_f = p^2 \tag{34}$$

$$B_f^* = 2p \tag{35}$$

$$B_c^* = \frac{3 + p^2}{2p} = 2p \tag{36}$$

$$L_c^* = \frac{3 - p^2}{2} = p^2 \tag{37}$$

Let's solve for p in L_c:

$$L_c^* = 3 - p^2 = 2p^2 \Longrightarrow 3 = 3p^2 \tag{38}$$

$$p^* = 1 \tag{39}$$

$$p^* = 1 \tag{40}$$

The resulting allocation is $L^* = 1$ hours of leisure and $B^* = 2$ pints of beer.

$$L_f = (1)^2 = 1 \tag{41}$$

$$B_f^* = 2 \cdot 1 = 2 \tag{42}$$

Question 3.a

2. Consider a Jekyll/Hyde economy in which Hyde has T hours of work available. There are two goods ale and bread. Jekyll can produce both goods with a one-to-one relation (one hour of labour can produce one unit of ale or one unit of bread). Hyde's preferences are summarised by the utility function:

$$U(a, b) = a + 2\sqrt{2b} \tag{43}$$

where a and b respectively denote the units of ale and bread Hyde consumes. Denotes the prices for the goods as p_a and p_b.

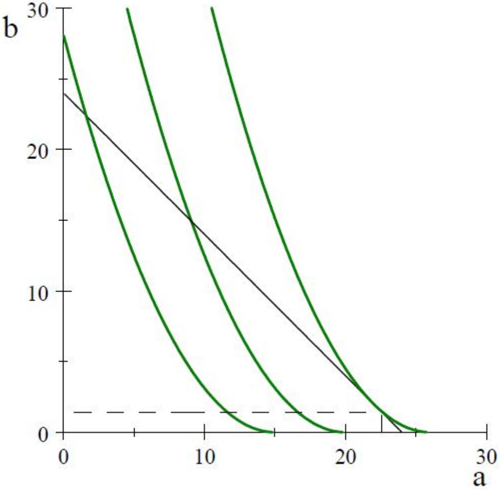
a. Describe graphically the equilibrium of this economy (place ale in the horizontal axis).

Because $a = L_a$ and $b = L_b$; the expression for the PPF is just

$$a + b = T \tag{44}$$

$$b = T - a \tag{45}$$

Graphically, the equilibrium occurs when the PPF is tangent to the consumer's indifference curve. See below the graph for the case T = 24.


b
30
20
10
0
0
10
20
30
a

Question 3.b

b. Find analytical the price ratio which constitutes a general equilibrium of this economy and the resulting allocation.

Analytically, once you have the MRPT (that is simply equal to - 1) yu need the MRS:

$$MU_a = 1 \tag{46}$$

$$MU_b = \sqrt{\frac{2}{b}} \tag{47}$$

$$MRS = \frac{MU_a}{MU_b} = \frac{1}{\sqrt{\frac{2}{b}}} \tag{48}$$

$$MRS = \sqrt{\frac{b}{2}} \tag{49}$$

Hence in equilibrium we know that:

$$MRPT = \frac{p_a}{p_b} = MRS \tag{50}$$

$$1 = \frac{p_a}{p_b} = \sqrt{\frac{b}{2}} \tag{51}$$

From that we can derive that in equilibrium $\frac{p_a}{p_b} = 1$ and that $b^* = 2$. Hence, $a^* = T - 2$. Notice that the fact that we can pin down the consumption of bread so easily is due to the fact that Hyde's preferences are quasi-linear so his consumption of bread is not subject to an income effect.

Question 4.a

4. Suppose that the production of x and y depends only on labour and the production functions are:

$$\begin{aligned} y &= f(l_y) = l_y^{0.5} \\ x &= f(l_x) = l_x^{0.5} \end{aligned} \tag{52}$$

And labour supply is fixed at $l_x + l_y = 100$.

a. Find the mathematical expression for the PPF. Is it linear or not? Why?

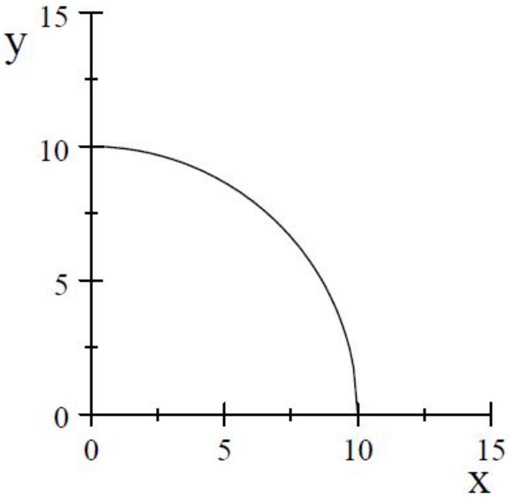
We can invert the goods' production function and so and out how much labour is needed to produce a certain amount of them.

$$y = l_y^{0.5} \Longrightarrow l_y = y^2 \tag{53}$$

$$x = l_x^{0.5} \Longrightarrow l_x = x^2 \tag{54}$$

Hence., since we have 100 units of labour available, the PPF (production production frontier) is given by

$$x^2 + y^2 = 100 \Longrightarrow y = \sqrt{100 - x^2} \tag{55}$$



The PPF is not linear because labour has diminishing returns in the production of both goods.

Question 4.b

b. Find the MRPT

The MRPT is just the slope of the PPF, hence:

$$y = \sqrt{100 - x^2} \tag{56}$$

$$MRST = \frac{\partial y}{\partial x} = \frac{\partial \sqrt{100 - x^2}}{\partial x} \tag{57}$$

$$MRST = \frac{\partial y}{\partial x} = \frac{\partial \sqrt{100 - x^2}}{\partial x} \frac{\partial - x^2}{\partial x} \tag{58}$$

$$MRST = \frac{x}{\sqrt{100 - x^2}} \tag{59}$$

Question 4.c

Suppose preferences of the consumer are represented by the utility function $U(x, y) = xy$. What is the equilibrium price ratio?

We know that the MRS is equal to the ratio of the MUs:

$$U(x, y) = xy \tag{60}$$

$$MU_x = \frac{\partial U(x, y)}{\partial x} = y \tag{61}$$

$$MU_y = \frac{\partial U(x, y)}{\partial y} = x \tag{62}$$

$$MRS = \frac{y}{x} \tag{63}$$

In equilibrium it must be that:

$$MRS = \frac{p_x}{p_y} = MRPT \tag{64}$$

$$\frac{y}{x} = \frac{p_x}{p_y} = \frac{x}{\sqrt{100 - x^2}} \tag{65}$$

Then, in equilibrium it must be that:

$$\frac{y}{x} = \frac{x}{\sqrt{100 - x^2}} \tag{66}$$

$$y = \frac{x^2}{\sqrt{100 - x^2}} \tag{67}$$

But this output combination must also belong to the PPF,hence:

$$\frac{x^2}{\sqrt{100-x^2}} = \sqrt{100-x^2} \tag{68}$$

$$x^2 = \sqrt{100-x^2} \cdot \sqrt{100-x^2} \tag{69}$$

$$x^2 = 100 - x^2 \implies x^2 + x^2 = 100 \tag{70}$$

$$2x^2 = 100 \tag{71}$$

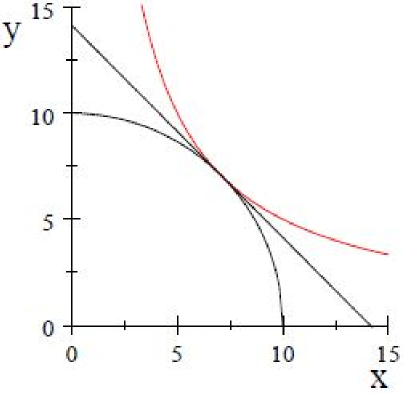
$$x = 5\sqrt{2} \text{ and } y = 5\sqrt{2} \tag{72}$$

So the equilibrium price ratio must be equal to:

$$\frac{p_x}{p_y} = \frac{5\sqrt{2}}{5\sqrt{2}} \tag{73}$$

$$\frac{p_x}{p_y} = 1 \tag{74}$$

Graphically the equilibrium looks like this:



Question 5.a

5. Suppose that the production possibility frontier can be represented by $x^2 + y^2 = 1$ and that Jekyll/Hyde's preferences can be represented by $U(x, y) = x^{0.5}y^{0.5}$.

a. Find the profit-maximising condition given p_x and p_y.

For the firm to maximise profits, the MRPT must be equal to the ratio of prices. The expression for the PPF is simply

$$y = \sqrt{1 - x^2} \tag{75}$$

Hence, the MRPT:

$$y = \sqrt{1 - x^2} \Longrightarrow \frac{\partial y}{\partial x} = \frac{-x}{\sqrt{1 - x^2}} \tag{76}$$

Thus, $MRPT = \frac{x}{\sqrt{1-x^2}}$

$$\frac{x}{\sqrt{1 - x^2}} = \frac{p_x}{p_y} \tag{77}$$

Question 5.b

5.b Find the utility-maximising condition given p_x and p_y.

Similarly, the consumer maximises utility when:

$$MRS = \frac{MU_x}{MU_y} = \frac{p_x}{p_y} \tag{78}$$

$$MU_x = \frac{\partial U}{\partial x} = \frac{x^{0.5}y^{0.5}}{\partial x} = 0.5y^{0.5}x^{\frac{1}{2}-1} \tag{79}$$

$$MU_x = \frac{\sqrt{y}}{2\sqrt{x}} \tag{80}$$

If you do the MU_y,you will get:

$$MU_y = \frac{\sqrt{x}}{2\sqrt{y}} \tag{81}$$

Then, the MRS is equal to:

$$MRS = \frac{\frac{\sqrt{y}}{2\sqrt{x}}}{\frac{\sqrt{x}}{2\sqrt{y}}} = \frac{y}{x} = \frac{p_x}{p_y} \tag{82}$$

Question 5.c

c. Find the relationship between the consumption of x and y in the competitive equilibrium of this economy

In the competitive equilibrium of this economy

$$MRS = \frac{p_x}{p_y} = MRPT \tag{83}$$

Then, the MRPT is equal to:

$$\frac{y}{x} = \frac{x}{\sqrt{1-x^2}} \tag{84}$$

$$\frac{y}{x} = \frac{x}{\sqrt{1-x^2}} \Longrightarrow y = \frac{x^2}{\sqrt{1-x^2}} \tag{85}$$

As we did in question 3.c, we need to this output combination must also belong to the PPF, hence:

$$y = \sqrt{1 - x^2} \tag{86}$$

$$\frac{x^2}{\sqrt{1 - x^2}} = \sqrt{1 - x^2} \implies x^2 = \sqrt{1 - x^2} \cdot \sqrt{1 - x^2} \tag{87}$$

$$x^2 = 1 - x^2 \implies 2x^2 = 1 \implies x^2 = \frac{1}{2} \tag{88}$$

$$x^* = \frac{1}{\sqrt{2}} \tag{89}$$

To find y^*, we can plug into the PPF:

$$y = \sqrt{1 - x^2} \implies y = \sqrt{1 - (\frac{1}{\sqrt{2}})^2} \implies y^* = \frac{1}{\sqrt{2}} \tag{90}$$

Then, the price ratios is equal to 1:

$$\frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = \frac{p_x}{p_y} = 1 \tag{91}$$

6. Suppose that country A and country B both produce wine and cheese. Country A has 800 units of available labour, while country B has 600 units. Prior to trade, country A consumes 40 kilos of cheese and 8 bottles of wine, and country B consumes 30 kilos of cheese and 10 bottles of wine.

	Country A	Country B
Labour per kg of cheese	10	10
Labour per bottle of wine	50	30

a. Which country has a comparative advantage in the production of each good? Explain

We need to look at the MRPTs of each country to identify which country has the competitive advantage on each good:

$$MRPT_A = \frac{10}{50} = -\frac{1}{5} \tag{92}$$

$$MRPT_B = \frac{10}{30} = -\frac{1}{3} \tag{93}$$

For country A the MRPT is just $\frac{1}{5}$ (it has to give up 5 kilos of cheese to produce one more bottle of wine), whereas the MRPT for country B is $\frac{1}{3}$ (it has to give up 3 kilos of cheese to produce one more bottle of wine). Hence, Country A has a comparative advantage in the production of cheese. And Country B in the production of wine.

Question 6.b

b. Determine the production possibilities frontier for each country, both graphically and algebraically (place cheese in the horizontal axis). Label the pre-trade production point as P. Label as T the point in their PPF each country will choose to produce after opening to trade.

For Country A:

$$L_W^A + L_C^A = 800 \tag{94}$$

$$L_W^A = 50W^A \tag{95}$$

$$L_C^A = 10C^A \tag{96}$$

Hence simply:

$$50W^A + 10C^A = 800 \Longrightarrow W^A = 16 - \frac{C^A}{5} \tag{97}$$

You can see now more clearly that the MRPT is equal to $\frac{1}{5}$

For Country B:

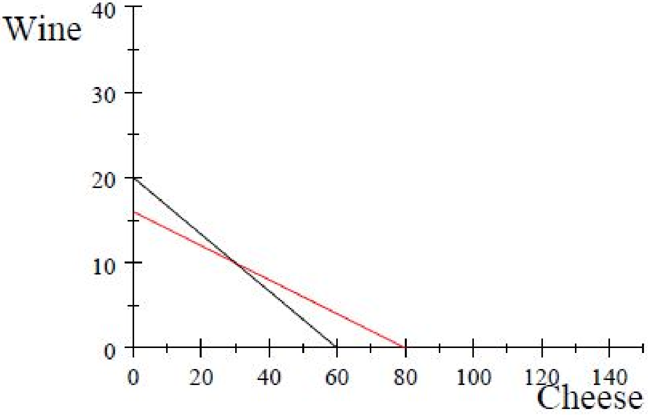
$$L_W^B + L_C^B = 600 \tag{98}$$

$$L_W^B = 30W^B \tag{99}$$

$$L_C^A = 10C^B \tag{100}$$

Hence simply:

$$30W^B + 10C^B = 800 \Longrightarrow W^B = 20 - \frac{C^B}{3} \tag{101}$$



Country A —— and Country B - - -

Question 6.c

c. Find the world PPF. Given that 36 kilos of cheese and 9 bottles of wine are traded, label the post-trade consumption points for each country. Are the countries better off as a result of opening to trade?

Country A has comparative advantage in the production of cheese. Hence it should be the first in production cheese. The equation for the PPF is then:

$$W = \begin{cases} 36 - \frac{C}{5} \text{ if } C \leq 80 \\ \frac{140}{3} - \frac{C}{3} \text{ if } C > 80 \end{cases} \tag{102}$$

Country A has comparative advantage in the production of cheese. Hence it should be the first in production cheese. The equation for the PPF is then:

$$W = \begin{cases} 36 - \frac{C}{5} \text{ if } C \leq 80 \\ \frac{140}{3} - \frac{C}{3} \text{ if } C > 80 \end{cases} \tag{103}$$

When C = 0 for country A, W = 16. For Country B, when C = 0, W = 20. Then, the intercept is 20 + 16 = 36.

Let's see how do get the these limits. From previously:

$$W^A = 16 - \frac{C}{5} \implies 0 = 16 - \frac{C}{5} \implies C = 80 \tag{104}$$

Now we have 36 kilograms of cheese, thus our new PPF is the following:

$$W = 36 - \frac{C}{5} \tag{105}$$

For the second part of the PPF:

$$20 = C_{int} - \frac{1}{3} \cdot 80^{1} \tag{106}$$

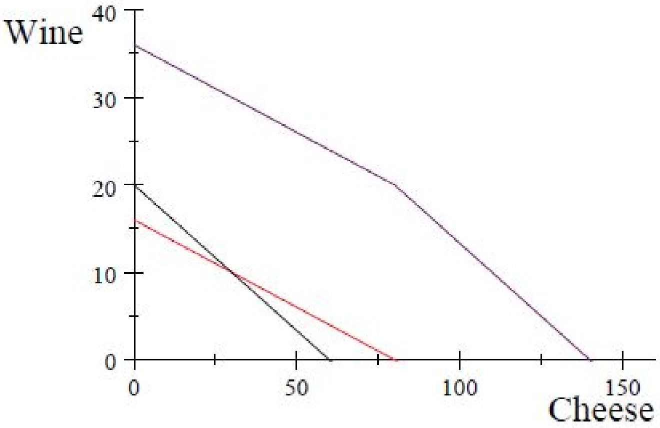
$$20 + \frac{1}{3} \cdot 80 = C_{int} \tag{107}$$

$$20 + \frac{1}{3} \cdot 80 = C_{int} \Longrightarrow \frac{60 + 80}{3} = C_{int} \Longrightarrow \frac{140}{3} = C_{int} \tag{108}$$

Thus:

$$W = \frac{140}{3} - \frac{C}{3} \text{ for } 80 > C \tag{109}$$

[1] 20 comes from the value when country b only produces cheese.


Wine
40
30
20
10
0
0
50
100
150
Cheese

Country A specialises in cheese so its production is represented by the point (80,0) whereas country B specialises in wine so its production is represented by the point (0,20). If 36 kilos of cheese and 9 bottles of wine are traded, it means that Country A is now consuming 44 kilos of cheese and 9 bottles of wine, whereas country B is consuming 36 kilos of cheese and 11 bottles of wine.

Consumption before trade:

- 40 kilos of cheese, Country A
- 8 bottles of wine, Country A
- 30 Kilos of Cheese, Country B
- 10 bottles of wines, Country B

With trade:

- 44 kilos of cheese, Country A (80-36)
- 9 bottles of wine, Country A
- 36 Kilos of Cheese, Country B
- 11 bottles of wines, Country B (11-9)