SEMINAR 1 - 6SSPP383

Formal Models of Political Economy - Electoral Rules

06-10-2023

EXERCISE 1

EXERCISE 1

- Consider a situation with a popular jury of 15 members. They have to decide whether a defendant is guilty of 1st degree murder (option a), 2nd degree murder (option b), or innocent (option c). The below table summarises preferences of the 15 judges:

4	2	5	4
a	c	b	c
b	a	a	b
c	b	c	a

Jury's preferences

(a) what is the veredict under plurality rule and under Borda count?

EXERCISE 1

- Can anyone explain how *plurality rule* works?

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Plurality rule: Choose the candidate who is ranked first by the largest number of voters (Mueller, pp 147)

EXERCISE 1 - PLURALITY

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4	2	5	4
a	c	b	c
b	a	a	b
c	b	c	a

Jury's preferences

EXERCISE 1 - PLURALITY

- *a* gets 4 votes

4	2	5	4
a	c	b	c
b	a	a	b
c	b	c	a

Jury's preferences

EXERCISE 1 - PLURALITY

- *a* gets 4 votes
- *b* gets 5 votes

4	2	5	4
a	c	b	c
b	a	a	b
c	b	c	a

Jury's preferences

EXERCISE 1 - PLURALITY

- *a* gets 4 votes
- *b* gets 5 votes
- *c* gets 6 votes

4	2	5	4
a	c	b	c
b	a	a	b
c	b	c	a

Jury's preferences

With *Plurality* (under sincere voting) *c* wins and hence the defendant is set free.

EXERCISE 1

- Can anyone explain how *Borda count* works?

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- Can anyone explain how *Borda count* works?

Borda count: Give each of the *m* candidates a score of 1 to *m* based on the candidate's ranking in a voter's preference ordering; that is, the candidate ranked first receives *m* points, the second one *m* – 1 points..., the lowest ranked one point. The candidate with the highest number of point is declared the winner (Mueller, pp 148).

So *m* **should be 3** but in the solutions *m* **is set equal to 2**. I believe this was done to simplify the calculation, but you should reach the same solution.

EXERCISE 1 - BORDA COUNT

Points	4	2	5	4
2	a	c	b	c
1	b	a	a	b
0	c	b	c	a

Jury's preferences

EXERCISE 1 - BORDA COUNT

- *a* (1st degree murder): 4(2) + 2(1) + 5(1) + 4(0) = 15

Points	4	2	5	4
2	a	c	b	c
1	b	a	a	b
0	c	b	c	a

Jury's preferences

EXERCISE 1 - BORDA COUNT

- *a* (1st degree murder): 4(2) + 2(1) + 5(1) + 4(0) = 15
- *b* (2nd degree murder): 4(1) + 2(0) + 2(5) + 4(1) = 18

Points	4	2	5	4
2	a	c	b	c
1	b	a	a	b
0	c	b	c	a

Jury’s preferences

EXERCISE 1 - BORDA COUNT

- *a* (1st degree murder): 4(2) + 2(1) + 5(1) + 4(0) = 15
- *b* (2nd degree murder): 4(1) + 2(0) + 2(5) + 4(1) = 18
- *c* (not guilty): 4(0) + 2(2) + 2(0) + 4(2) = 12

With *Borda*, *b* wins, which is 2nd degree murder.

Points	4	2	5	4
2	a	c	b	c
1	b	a	a	b
0	c	b	c	a

Jury's preferences

EXERCISE 1

What is the trade-off in using either method?

EXERCISE 1

What is the trade-off in using either method?

1. With *Borda* the intensity of preferences is taken into account. With *Plurality* is not. Plus, with *Borda*, we incorporate voters' whole set of individual preferencing order into the voting mechanism.
2. *Plurality* is a much simpler voting mechanism.

EXERCISE

Which method is sounds fairer in this context and why?

EXERCISE

Which mechanism yields a "fairer" result?

- 11 jurors prefer 1st degree murder as an outcome (1st or 2nd preference)
- 13 jurors prefer 2nd degree murder as an outcome (1st or 2nd preference)
- Whereas 6 jurors prefer innocent as an outcome (1st or 2nd preference)

Plurality will not yield a "fair" result (Innocent). Whereas *Borda* rule would yield a "fair" result (2nd degree murder).

EXERCISE 2

EXERCISE 2

Consider a situation with three voters and the choice set $X = \{a, b, c\}$. Voters have the following preferences:

$$\begin{aligned} 1: &\quad a\,P\,b\,P\,c \\ 2: &\quad b\,P\,c\,P\,a \\ 3: &\quad c\,P\,a\,P\,b \end{aligned}$$

Is there a Condorcet winner?

Consider a situation with three voters and the choice set $X = \{a, b, c\}$. Voters have the following preferences:

$$\begin{aligned} 1: &\quad a \, P \, b \, P \, c \\ 2: &\quad b \, P \, c \, P \, a \\ 3: &\quad c \, P \, a \, P \, b \end{aligned}$$

Is there a Condorcet winner?

- a vs b, 2 votes
- a vs c, 2 votes
- b vs c, 2 votes

Thus, there is no Condorcet winner There is no candidate (option) that defeats all others in pairwise elections using majority rule.

EXERCISE 2, A

To choose among three alternatives, the following voting rule is adopted: In the first stage, each voter cast one vote. If one option has the majority, the game ends and that option wins. Else, then, option c is arbitrarily eliminated and there is a runoff election between a and b.

(a) Find all the Nash equilibria of this assuming that voters vote *sincerely*.

EXERCISE 2, A

Before we obtain the NEs, let's see who is the number under this voting scheme, if voters vote *sincerely*

$$\begin{aligned} 1: &\quad a\,P\,b\,P\,c \\ 2: &\quad b\,P\,c\,P\,a \\ 3: &\quad c\,P\,a\,P\,b \end{aligned}$$

(a) If voters vote *sincerely* - **First round**:

- a receives 1 vote
- b receives 1 vote
- c receives 1 vote

There is no majority, then, c is arbitrarily eliminated and there is a runoff between candidates a and b.

EXERCISE 2, A

If voters vote *sincerely* - **Second round**:

$$\begin{aligned} 1: &\quad a\,P\,b\,P\,c \\ 2: &\quad b\,P\,c\,P\,a \\ 3: &\quad c\,P\,a\,P\,b \end{aligned}$$

- a receives 2 votes (1 & 3)
- b receives 1 vote (only 2)

Then, candidate a wins election under this voting scheme $\{a, b, a\}$.

EXERCISE 2, A - PAYOFFS

		Player 3								
		a			b			c		
		Player 2								
		a	b	c	a	b	c	a	b	c
Player 1	a									
	b									
	c									

EXERCISE 2, A - PAYOFFS

$$1: \quad a\,P\,b\,P\,c$$

$$2: \quad b\,P\,c\,P\,a$$

$$3: \quad c\,P\,a\,P\,b$$

$$U_{1,a,S_2,S_3} = \begin{cases} 2 & \text{if } S_2 = a \text{ and } S_3 = a \\ 2 & \text{if } S_2 = b \text{ and } S_3 = a \\ 2 & \text{if } S_2 = c \text{ and } S_3 = a \\ 2 & \text{if } S_2 = a \text{ and } S_3 = b \\ 1 & \text{if } S_2 = b \text{ and } S_3 = b \\ 2 & \text{if } S_2 = c \text{ and } S_3 = b \\ 2 & \text{if } S_2 = a \text{ and } S_3 = c \\ 2 & \text{if } S_2 = b \text{ and } S_3 = c \\ 0 & \text{if } S_2 = c \text{ and } S_3 = c \end{cases} \quad (1)$$

EXERCISE 2, A - PAYOFFS

		Player 3								
		a			b			c		
		Player 2								
		a	b	c	a	b	c	a	b	c
Player 1	a	2, 0, 1	2, 0, 1	2, 0, 1	2, 0, 1	1, 2, 0	2, 0, 1	2, 0, 1	2, 0, 1	0, 1, 2
	b									
	c									

1st Round

EXERCISE 2, A - FIND NASH EQUILIBRIA

		Player 3								
		a			b			c		
		Player 2								
		a	b	c	a	b	c	a	b	c
Player 1	a	2, 0, 1	2, 0, 1	2, 0, 1	2, 0, 1	1, 2, 0	2, 0, 1	2, 0, 1	2, 0, 1	0, 1, 2
	b	2, 0, 1	1, 2, 0	2, 0, 1	1, 2, 0	1, 2, 0	1, 2, 0	2, 0, 1	1, 2, 0	0, 1, 2
	c	2, 0, 1	2, 0, 1	0, 1, 2	2, 0, 1	1, 2, 0	0, 1, 2	0, 1, 2	0, 1, 2	0, 1, 2

1st Round

EXERCISE 2, A - FIND NASH EQUILIBRIA

		Player 3								
		a			b			c		
		Player 2								
		a	b	c	a	b	c	a	b	c
Player 1	a	2, 0, 1	2, 0, 1	2, 0, 1	2, 0, 1	1, 2, 0	2, 0, 1	2, 0, 1	2, 0, 1	0, 1, 2
	b	2, 0, 1	1, 2, 0	2, 0, 1	1, 2, 0	1, 2, 0	1, 2, 0	2, 0, 1	1, 2, 0	0, 1, 2
	c	2, 0, 1	2, 0, 1	0, 1, 2	2, 0, 1	1, 2, 0	0, 1, 2	0, 1, 2	0, 1, 2	0, 1, 2

1st Round

EXERCISE 2, A - FIND NASH EQUILIBRIA

		Player 3								
		a			b			c		
		Player 2								
		a	b	c	a	b	c	a	b	c
Player 1	a	2, 0, 1	2, 0, 1	2, 0, 1	2, 0, 1	1, 2, 0	2, 0, 1	2, 0, 1	2, 0, 1	0, 1, 2
	b	2, 0, 1	1, 2, 0	2, 0, 1	1, 2, 0	1, 2, 0	1, 2, 0	2, 0, 1	1, 2, 0	0, 1, 2
	c	2, 0, 1	2, 0, 1	0, 1, 2	2, 0, 1	1, 2, 0	0, 1, 2	0, 1, 2	0, 1, 2	0, 1, 2

1st Round

EXERCISE 2, A - FIND NASH EQUILIBRIA

		Player 3								
		a			b			c		
		Player 2								
		a	b	c	a	b	c	a	b	c
Player 1	a	$\underline{2}, \underline{0}, \underline{1}$	$\underline{2}, \underline{0}, \underline{1}$	$\underline{2}, \underline{0}, 1$	$\underline{2}, 0, 1$	$\underline{1}, \underline{2}, 0$	$\underline{2}, 0, 1$	$\underline{2}, 0, \underline{1}$	$\underline{2}, 0, \underline{1}$	$\underline{0}, \underline{1}, \underline{2}$
	b	$\underline{2}, 0, \underline{1}$	$1, \underline{2}, \underline{0}$	$\underline{2}, 0, 1$	$1, \underline{2}, 0$	$\underline{1}, \underline{2}, \underline{0}$	$\underline{1}, \underline{2}, 0$	$\underline{2}, 0, \underline{1}$	$1, \underline{2}, \underline{0}$	$\underline{0}, 1, \underline{2}$
	c	$\underline{2}, 0, 1$	$\underline{2}, 0, 1$	$0, \underline{1}, \underline{2}$	$\underline{2}, 0, 1$	$\underline{1}, \underline{2}, 0$	$0, 1, \underline{2}$	$0, \underline{1}, \underline{2}$	$0, 1, 2$	$\underline{0}, \underline{1}, \underline{2}$

1st Round

$\{a, a, a\}$, $\{a, b, a\}$, $\{b, b, b\}$, $\{a, c, c\}$ and $\{c, c, c\}$ are the NE.

EXERCISE 2, B

To choose among three alternatives, the following voting rule is adopted: In the first stage, each voter cast one vote. If one option has the majority, the game ends and that option wins. Else, then, option *c* is arbitrarily eliminated and there is a runoff election between *a* and *b*.

(b) Suppose voters vote *strategically* instead. Start from the 2nd stage. First, find all the Nash equilibria. Then, eliminate Nash equilibria in which players adopt weakly dominated strategies.

EXERCISE 2, A - PAYOFFS

1 : $a \, P \, b \, P \, c$

2 : $b \, P \, c \, P \, a$

3: $c \, P \, a \, P \, b$

		Player 3			
		a		b	
		Player 2			
		a	b	a	b
Player 1	a				
	b				

EXERCISE 2, A

		Player 3			
		a		b	
		Player 2			
		a	b	a	b
Player 1	a	2, 0, 1	2, 0, 1	2, 0, 1	1, 2, 0
	b				

EXERCISE 2, A

		Player 3			
		a		b	
		Player 2			
		a	b	a	b
Player 1	a	2, 0, 1	2, 0, 1	2, 0, 1	1, 2, 0
	b	2, 0, 1	1, 2, 0	1, 2, 0	1, 2, 0

EXERCISE 2, A - PLAYER 1'S BRS

		Player 3			
		a		b	
		Player 2			
		a	b	a	b
Player 1	a	$\underline{2}$, 0, 1	$\underline{2}$, 0, 1	$\underline{2}$, 0, 1	$\underline{1}$, 2, 0
	b	$\underline{2}$, 0, 1	1, 2, 0	1, 2, 0	$\underline{1}$, 2, 0

EXERCISE 2, A PLAYER 2'S BRS

		Player 3			
		a		b	
		Player 2			
		a	b	a	b
Player 1	a	$\underline{2}$, $\underline{0}$, 1	$\underline{2}$, $\underline{0}$, 1	$\underline{2}$, 0, 1	$\underline{1}$, $\underline{2}$, 0
	b	$\underline{2}$, 0, 1	1, $\underline{2}$, 0	1, $\underline{2}$, 0	$\underline{1}$, $\underline{2}$, 0

EXERCISE 2, A PLAYER 3'S BRS

		Player 3			
		a		b	
		Player 2			
		a	b	a	b
Player 1	a	$\underline{2}, \underline{0}, \underline{1}$	$\underline{2}, 0, \underline{1}$	$\underline{2}, \underline{0}, \underline{1}$	$\underline{1}, \underline{2}, 0$
	b	$\underline{2}, 0, \underline{1}$	$1, \underline{2}, \underline{0}$	$1, \underline{2}, 0$	$\underline{1}, \underline{2}, \underline{0}$

EXERCISE 2, A

How many NE equilibria there are? and which one they are?

		Player 3			
		a		b	
		Player 2			
		a	b	a	b
Player 1	a	$\underline{2}, \underline{0}, \underline{1}$	$\underline{2}, 0, \underline{1}$	$\underline{2}, \underline{0}, \underline{1}$	$\underline{1}, \underline{2}, 0$
	b	$\underline{2}, 0, \underline{1}$	$1, \underline{2}, \underline{0}$	$1, \underline{2}, 0$	$\underline{1}, \underline{2}, \underline{0}$

EXERCISE 2, A

		Player 3			
		a		b	
		Player 2			
		a	b	a	b
Player 1	a	2, 0, 1	2, 0, 1	2, 0, 1	1, 2, 0
	b	2, 0, 1	1, 2, 0	1, 2, 0	1, 2, 0

Three NE: {aaa}, {aba}, {bbb}

EXERCISE 2, REMOVING WEAKLY DOMINATED STRATEGIES FOR PLAYER 1

		Player 3			
		a		b	
		Player 2			
		a	b	a	b
Player 1	a	2, 0, 1	2, 0, 1	2, 0, 1	1, 2, 0
	b	2, 0, 1	1, 2, 0	1, 2, 0	1, 2, 0

Does player 1 have a weakly dominated strategy?

Caveat: The order at which delete weakly dominated strategies affects the final outcome. So, the answer to the question above is not complete. One should specify the order of deleting. Here we started with player 1, then 2, and then 3. **In this case, the order does not matter.**

EXERCISE 2, A - REMOVING WEAKLY DOMINATED STRATEGIES FOR PLAYER 1

		Player 3			
		a		b	
		Player 2			
		a	b	a	b
Player 1	a	2, 0, 1	2, 0, 1	2, 0, 1	1, 2, 0

Does player 1 have a weakly dominated strategy? *b* is weakly dominated by *a*

EXERCISE 2, A - REMOVING WEAKLY DOMINATED STRATEGIES FOR PLAYER 2

		Player 3			
		a		b	
		Player 2			
		a	b	a	b
Player 1	a	2, 0, 1	2, 0, 1	2, 0, 1	1, 2, 0

Does player 2 have a weakly dominated strategy?

EXERCISE 2, A - REMOVING WEAKLY DOMINATED STRATEGIES FOR PLAYER 2

		Player 3	
		a	b
		Player 2	
		b	b
Player 1	a	2, 0, 1	1, 2, 0

Does player 2 have a weakly dominated strategy? *a* is weakly dominated by *b*

EXERCISE 2, A- REMOVING WEAKLY (STRICTLY) DOMINATED STRATEGIES FOR PLAYER 3

		Player 3	
		a	b
		Player 2	
		b	b
Player 1	a	2, 0, 1	1, 2, 0

Does player 3 have a weakly (strictly) dominated strategy?

EXERCISE 2, A - REMOVING WEAKLY DOMINATED STRATEGIES FOR PLAYER 3

		Player 3
		a
		Player 2
		b
Player 1	a	2, 0, 1

Does player 3 have a weakly (strictly) dominated strategy? *a* is weakly dominated by *b*

EXERCISE 2, A - REMOVING WEAKLY DOMINATED STRATEGIES FOR PLAYER 3

		Player 3
		a
		Player 2
		b
Player 1	a	2, 0, 1

Thus, if we delete *weakly dominated strategies*, then the only equilibrium is {*a*, *b*, *a*}. Player *b* is weakly dominated for Player 1 and Player 3, while *a* is so for Player 2.

EXERCISE 2, C

(c) Building from above solution, solve the 1st stage in the same way: First, find all the Nash equilibra. Then, eliminate Nash equilibria in which players adopt weakly dominated strategies.

EXERCISE 2, C

We find all NE in Exercise 1,a.

		Player 3								
		a			b			c		
		Player 2								
		a	b	c	a	b	c	a	b	c
Player 1	a	$\underline{2}, \underline{0}, \underline{1}$	$\underline{2}, \underline{0}, \underline{1}$	$\underline{2}, \underline{0}, 1$	$\underline{2}, 0, 1$	$\underline{1}, \underline{2}, 0$	$\underline{2}, 0, 1$	$\underline{2}, 0, \underline{1}$	$\underline{2}, 0, \underline{1}$	$\underline{0}, \underline{1}, \underline{2}$
	b	$\underline{2}, 0, \underline{1}$	$1, \underline{2}, \underline{0}$	$\underline{2}, 0, 1$	$1, \underline{2}, 0$	$\underline{1}, \underline{2}, \underline{0}$	$\underline{1}, \underline{2}, 0$	$\underline{2}, 0, \underline{1}$	$1, \underline{2}, \underline{0}$	$\underline{0}, 1, \underline{2}$
	c	$\underline{2}, 0, 1$	$\underline{2}, 0, 1$	$0, \underline{1}, \underline{2}$	$\underline{2}, 0, 1$	$\underline{1}, \underline{2}, 0$	$0, 1, \underline{2}$	$0, \underline{1}, \underline{2}$	$0, 1, 2$	$\underline{0}, \underline{1}, \underline{2}$

1st Round

$\{a, a, a\}$, $\{a, b, a\}$, $\{b, b, b\}$, $\{a, c, c\}$ and $\{c, c, c\}$ are the NE.

EXERCISE 2, C - PAYOFFS

		Player 3								
		a			b			c		
		Player 2								
		a	b	c	a	b	c	a	b	c
Player 1	a	2, 0, 1	2, 0, 1	2, 0, 1	2, 0, 1	1, 2, 0	2, 0, 1	2, 0, 1	2, 0, 1	0, 1, 2
	b	2, 0, 1	1, 2, 0	2, 0, 1	1, 2, 0	1, 2, 0	1, 2, 0	2, 0, 1	1, 2, 0	0, 1, 2
	c	2, 0, 1	2, 0, 1	0, 1, 2	2, 0, 1	1, 2, 0	0, 1, 2	0, 1, 2	0, 1, 2	0, 1, 2

1st Round

EXERCISE 2, C - REMOVING WEAKLY DOMINATED STRATEGIES FOR PLAYER 1

		Player 3								
		a			b			c		
		Player 2								
		a	b	c	a	b	c	a	b	c
Player 1	a	2, 0, 1	2, 0, 1	2, 0, 1	2, 0, 1	1, 2, 0	2, 0, 1	2, 0, 1	2, 0, 1	0, 1, 2

1st Round - Removing weakly dominated strategies

EXERCISE 2, C - REMOVING WEAKLY DOMINATED STRATEGIES FOR PLAYER 2

		Player 3								
		a			b			c		
		Player 2								
		a	b	c	a	b	c	a	b	c
Player 1	a	2, 0, 1	2, 0, 1	2, 0, 1	2, 0, 1	1, 2, 0	2, 0, 1	2, 0, 1	2, 0, 1	0, 1, 2

1st Round - Removing weakly dominated strategies

Does player 2 have a weakly dominated strategy?

EXERCISE 2, C - REMOVING WEAKLY DOMINATED STRATEGIES FOR PLAYER 2

		Player 3								
		a			b			c		
		Player 2								
		a	b	c	a	b	c	a	b	c
Player 1	a	2, 0, 1	2, 0, 1	2, 0, 1	2, 0, 1	1, 2, 0	2, 0, 1	2, 0, 1	2, 0, 1	0, 1, 2

1st Round - Removing weakly dominated strategies

Does player 2 have a weakly dominated strategy? We can see that *a* is weakly dominated by *b*. However, *c* is not weakly dominated. If player 3, chooses *b*, player 2's BR is *b*, but if player 3 chooses *c*, player 2's BR is *c*.

EXERCISE 2, C - REMOVING WEAKLY DOMINATED STRATEGIES FOR PLAYER 2

		Player 3					
		a		b		c	
		Player 2					
		b	c	b	c	b	c
Player 1	a	2, 0, 1	2, 0, 1	1, 2, 0	2, 0, 1	2, 0, 1	0, 1, 2

1st Round - Removing weakly dominated strategies

Hence, we can delete candidate *a* from player 2.

EXERCISE 2, C - REMOVING WEAKLY DOMINATED STRATEGIES FOR PLAYER 3

		Player 3					
		a		b		c	
		Player 2					
		b	c	b	c	b	c
Player 1	a	2, 0, 1	2, 0, 1	1, 2, 0	2, 0, 1	2, 0, 1	0, 1, 2

1st Round - Removing weakly dominated strategies

Are there any weakly dominated strategies for player 3?

EXERCISE 2, C - REMOVING WEAKLY DOMINATED STRATEGIES FOR PLAYER 3

		Player 3					
		a		b		c	
		Player 2					
		b	c	b	c	b	c
Player 1	a	2, 0, 1	2, 0, 1	1, 2, 0	2, 0, 1	2, 0, 1	0, 1, 2

1st Round - Removing weakly dominated strategies

Are there any weakly dominated strategies for player 3? If Player 2 chooses *b*, Player 3's BR is either *b* or *c*. If Player 2, chooses *c*, Player 3's BR is choosing *c*, thus, *a* and *b* and weakly dominated by *c* for Player 3.

EXERCISE 2, A - REMOVING WEAKLY DOMINATED STRATEGIES FOR PLAYER 3

		Player 3
		Player 2
		c
Player 1	a	0, 1, 2

1st Round - Removing weakly dominated strategies

{*a*, *c*, *c*} is the only NE that survives. We then find that *a* and *b* are weakly dominated by *c* for player 3.

Conclusion {*a*, *c*, *c*} is the NE that survives.

EXERCISE 2, C - REMOVING WEAKLY DOMINATED STRATEGIES FOR PLAYER 3

		Player 3
		Player 2
		c
Player 1	a	0, 1, 2

1st Round - Removing weakly dominated strategies

Conclusion $\{a, c, c\}$ is the NE that survives. Candidates c wins the election.

EXERCISE 2, C - CONCLUSION

- In the first setup, **candidate *a* wins the election**, and candidate ***c*** is dropped from the ballot.
- But if voters vote *strategically*, **candidate *c* wins the election**.
- The main take away from this exercise is that, under certainty conditions, some voting rules may backfire.

SEMINAR 2

6SSPP383 - Formal models of Political Economy: May’s Theorem

06-10-2023

PROOF MAY’S THEOREM

MAY'S THEOREM

Disclaimer: I largely borrowed from Professor Dr. Robert Powers's proof slides on the May's Theoreom.

MAY’S THEOREM

Definition: In an election with two candidates, a social choice function that is anonymous, neutral, monotone, and nearly decisive is (functionally equivalent to) the simple majority method.

NOTATION

N = 1, 2, ...n with n >= 2

Two alternatives: 1, and -1. Abstention voted as 0

$$F : \{-1, 0, 1\}^n \longrightarrow \{-1, 0, 1\} \tag{1}$$

These are the list of votes.

$$R = (R_1, ..., R_n) \in \{-1, 0, 1\}^n \tag{2}$$

When they are in the domain F, we call them **profiles**.

NOTATION

For any profile $R = (R_1.....R_n)$, we have three sets of voters:

$$N_+ = \{i \in N : R_i = 1\} \text{ and } N_- = \{i \in N : R_i = -1\} \text{ and } N_0 = \{i \in N : R_i = 0\} \quad (3)$$

In addition, we let:†

$$n_+(R) = |N_+(R)|, n_-(R) = |N_-(R)|, \text{ and } n_0(R) = |N_0(R)| \quad (4)$$

SIMPLE MAJORITY RULE

The rule $F_M : \{-1, 0.1\} \rightarrow \{-1, 0.1\}$ defined by:

$$F_M(R) = \begin{cases} -1 & \text{if } n_-(R) > n_+(R) \\ 0 & \text{if } n_-(R) = n_+(R) \\ 1 & \text{if } n_-(R) < n_+(R) \end{cases} \tag{5}$$

SIMPLE MAJORITY RULE

For any profile $R = (R_1, ...R_n)$, we let:

$$S_R = \sum_{i=1}^{n} R_i \tag{6}$$

We observe that :

$$S_R = n_+(R) - n_-(R) \tag{7}$$

SIMPLE MAJORITY RULE

Therefore, any profile $R \in \{-1, 0.1\}^n$

$$F_M(R) = \text{sign}(S_R) \tag{8}$$

$$\text{Sign} = \begin{cases} -1 & \text{if voters prefers -1 over 1} \\ 0 & \text{if there is a tie} \\ 1 & \text{if voters prefer 1 over -1} \end{cases} \tag{9}$$

AXIOMS

Anonymity (A). Given any $R = (R_1, ...R_n) \in \{-1, 0.1\}^n$ and any permutation $\sigma : N \to N$, we have $F(R) = F(R_\sigma)$ where $R_\sigma = (R_{\sigma(1)}, ..., R_{\sigma(n)})$
We don't have who cast the vote, we don't attach the vote. Now matter how we rearrange the votes, the output of the social function is the same.

AXIOMS

Monotonicity (M). For all $R, R' \in \{-1, 0, 1\}^n$, $F(R) \leq F(R')$ whenever $R \leq R'$ (i.e., $R_i \leq R'_i$ for $i = 1, \ldots, n$).

We are going rearrange profiles component-wise. We are going to say that R' is greater than or equal to R if for every single entry R_i, the corresponding entry in R'_i is greater or equal to the entry in R_i. When this happens $F(R) \leq F(R')$.

AXIOMS

Neutrality (N). $F(-R) = -F(R)$ for all $R \in \{-1, 0, 1\}^n$.

If voter *i* voted for 1, then her vote will shift to -1. Everyone's vote has flipped, and in that the case, the outcome should also flip. The motivation of this axiom is the labels attached to each candidate should not matter.

PROOF - CASE 1

We want to proof that $F = F_M$

Proof. Assume $F(R) : \{-1, 0, 1\}^n \to \{-1, 0, 1\}$ satisfies (A), (N) and (SM). Our goal is to show that $F = F_M$.

Let R be a profile where $S_R = 0$ and so $n_-(R) = n_+(R)$. Let $\sigma : N \to N$ be a permutation of N such that σ maps $N_+(R)$ onto $N_-(R)$ and maps onto $N_+(R)$. Observe that:

$$R_\sigma = -R \tag{10}$$

We are permuting the voting sets. Every positive is changing to negative ones.

PROOF - CASE 1

We use our axioms, we apply (A) first and then (N), we get:

$$R_\sigma = -R \tag{11}$$

$$\underbrace{F(R) = F(R_\sigma)}_{\text{Anonymity}} \tag{12}$$

$$F(R_\sigma) = \underbrace{F(-R) = -F(R)}_{\text{Neutrality}} \tag{13}$$

$F(R)$ and $-F(R)$ have to be equal. Since $F(R) \in \{-1, 0, 1\}$. Thus, the only way a number can be equal to its negative, is when it's exactly 0. Therefore; $F(R) = 0 = F_M(R)$.

Thus, simple majority rule outputs 0 when there is a voter tie $n_-(R) = n_+(R)$.

PROOF - CASE 2

Proof. Now suppose profile R satisfies $n_+(R) > n_-(R)$. So the number 1s is bigger than the number of -1s.

Choose a subset $\boldsymbol{I}$ of $n_+(R)$ such that $|\boldsymbol{I}| = n_+(R) - n_-(R)$. Next, let $Q = (Q_1, ..., Q_n)$ be the profile defined as follows:

$$Q_i = 0 \forall i \in \boldsymbol{I} \text{ and } Q_j = R_J \forall j \in \boldsymbol{N}\mathrm{I} \tag{14}$$

What we did here is we changed a bunch of 1s into 0s. So now the number of 1s and the number of -1s are the same.

PROOF - CASE 2

Thus $s(Q) = 0$ and $R > Q$. From the previous argument, $F(Q) = 0$. By strong monotonicity.

$$R > Q \text{ and } F(Q) = 0 \implies F(R) = 1 \tag{15}$$

Thus, this agrees with Majority Rule $F(R) = F_M(R)$ for any profile R such that $n_+(R) > n_-(R)$.

MAY’S THEOREM

Based on Mueller’s book

MAY'S THEOREM

1. Decisiveness: It always add to an integer, which by the decision function is transformed into -1, or 0, or +1, and thus is decisive.
2. Anonymity: Change any +1 or -1, and any -1 to +1, and then sum is left unchanged.
3. Neutrality: If *x* defeats (ties) *y* for one set of individual preferences , ad all individuals have the same *ordinal* rankings for *z* and *w* as for *x* and *y*, then *z* defeats *w*
4. Monotonicity: If $\sum_{i=1}^{n} D_i = 0$, increasing any D_i will make $\sum_{i=1}^{n} D_i > 0$, and decide the contest in favour of *x*. If $\sum_{i=1}^{n} D_i > 0$, increasing any D_i will leave $\sum_{i=1}^{n} D_i > 0$ and will not change the outcome.

Let's show the first three conditions imply:

$$[N(-1) = N(1)] \rightarrow 0 \tag{16}$$

$N(-1)$ is the number of voters that vote for and $N(1)$ is the number of voters that vote for x.

Let's assume that $[N(-1) = N(1)] \rightarrow 0$ does not hold, for example:

$$[N(-1) = N(1)] \rightarrow 1 \tag{17}$$

Let's assume that $[N(-1) = N(1)] \rightarrow 0$ does not hold, for example:

$$[N(-1) = N(1)] \rightarrow D = 1 \tag{18}$$

This means when the number of votes for y equals the number of votes for x, the outcome is x

1. **Step one:** Let's now relabel y to z and x to w, where a vote for z is now re-coded as –1 and a vote to w as a +1
2. **Step two:** Reverse all +1s to -1s and -1s to +1s. By anonymity the outcome is the same. $xR_i y$ now $zR_i w$

MAY'S THEOREM

Based on the neutrality axiom, the collective outcome must be z if it was originally x, but z is equivalent to y, not x. The decisiveness axiom is violated.

MAY'S THEOREM - CASE 2

$$[N(-1) = N(1)] \rightarrow D = -1 \tag{19}$$

Thus:

$$[N(-1) = N(1) + 1] \rightarrow D = +1 \tag{20}$$

When the number of votes for x is greater than y, then x must win.

Now, let's assume when the number of voters for x is $m - 1$ grater than the number for y, x wins.

MAY'S THEOREM - A MUCH SIMPLER PROOF (EXCLUDING MONOTONICITY

Three voters A, B, C

F(1, 0, -1) = 1, $F \neq F_M$. Majority rule will choose 0 $F_M(1, 0, -1) = 0$

F(-1, 0, 1) = 1 by anonymity, voters swap their votes.

F(1, 0, -1) = -1, by Neutrality, voters are not biased to a particular candidate.

But $F(1, 0, -1) \neq F(1, 0, -1)$, so F violates decisiveness.

Thus, no other social choice function meets all three axioms, just majority rule.

MAY’S THEOREM - A MUCH SIMPLER PROOF (INCLUDING MONONICITY

Three voters A, B, C

F(1, 1, -1) = -1 , $F \neq F_M$. Majority rule will choose $F_M(1, 1, -1) = 1$

F(-1, -1, 1) = 1 by Neutrality, voters are not biased to a particular candidate. The outcome

F(1, -1, -1) = 1, by Anonimity, voters swap their votes.

But $F(1, -1, -1) \neq F(1, 1, -1)$, so F violates decisiveness.

Thus, no other social choice function meets all three axioms, just majority rule.

SEMINAR 3

6SSPP383 - Formal models of political economy: Electoral rules II

20-10-2023

PROBLEM SET 3

FIFA (International Football Federation) decides the host country of the World Cup using a voting system of successive eliminations (i.e. in the first round the committee members cast one vote for their preferred alternative, and the least voted alternative is eliminated; then they vote among the remaining alternatives and the least voted alternative is eliminated, and so on and so forth).

PROBLEM SET 3

Assume that Nigeria, France, Argentina and the UK are the candidate countries to host the World Cup in 2030 and that the results of the voting process are given by the table below:

	1st Round	2nd Round	3er Round
Nigeria	77	81	89
France	53	65	121
Argentina	63	64	
UK	17		

Number of votes

The 210 members of FIFA decide by successive eliminations that the next host country of the world cup will be France.

PROBLEM SET 3 - I

	1st Round	2nd Round	3er Round
Nigeria	77	81	89
France	53	65	121
Argentina	63	64	
UK	17		

FIFA members’ preferences

i) How many FIFA members who have UK as their first choice, have Nigeria as their second choice? How many have France as their second choice? How many have Argentina as their second choice?

PROBLEM SET 3 - I

77	53	63	17
Nigeria	France	Argentina	UK
C2	C2	C2	C2
C3	C3	C3	C3
C4	C4	C4	C4

FIFA members' preferences

- Nigeria as a second choice: 81-77 = 4
- France as a second choice: 65-53 = 12
- Argentina as a second choice: 64-63 = 1

PROBLEM SET 3 - I

	1st Round	2nd Round	3er Round
Nigeria	77	81	89
France	53	65	121
Argentina	63	64	
UK	17		

FIFA members' preferences

- Nigeria as a second choice: 81-77 = 4
- France as a second choice: 65-53 = 12
- Argentina as a second choice: 64-63 = 1

PROBLEM SET 3 - II

ii) Which would be the winning country if FIFA used plurality rule instead?

	1st Round	2nd Round	3er Round
Nigeria	77	81	89
France	53	65	121
Argentina	63	64	
UK	17		

FIFA members' preferences

PROBLEM SET 3 - II

ii) Which would be the winning country if FIFA used plurality rule instead?

Plurality rule: Each voter cast a single vote for a single alternative, and the alternative with the most votes wins.

PROBLEM SET 3 - II

ii) Which would be the winning country if FIFA used plurality rule instead?

	1st Round	2nd Round	3er Round
Nigeria	77	81	89
France	53	65	121
Argentina	63	64	
UK	17		

FIFA members' preferences

Nigeria will win under plurality.

PROBLEM SET 3 - III

iii) Does the Condorcet winner coincide with the plurality winner?

	1st Round	2nd Round	3er Round
Nigeria	77	81	89
France	53	65	121
Argentina	63	64	
UK	17		

FIFA members' preferences

PROBLEM SET 3 - III

Condorcet method: There is one candidate (option) that defeats all others in pairwise elections using majority rule.

PROBLEM SET 3 - III

	1st Round	2nd Round	3er Round
Nigeria	77	81	89
France	53	65	121
Argentina	63	64	
UK	17		

FIFA members' preferences

We do not know who the Condorcet winner is. But we do know that it does not coincide with the plurality winner (Nigeria). This is so because in the last round loses in a pairwise vote from France, and the Condorcet winner is the alternative that wins in all pairwise voting procedures.

PROBLEM SET 3 - IV

iv) If we know that all members prefer UK to at least two other countries (from the four considered here), which voting rule would assign the world cup to UK?

PROBLEM SET 3 - IV - BORDA COUNT

Points	77	53	63	17
3	N	F	A	UK
2	UK	UK	UK	C2
1	C3	C3	C3	C3
0	C4	C4	C4	C4

FIFA Members’ preferences

PROBLEM SET 3 - IV - BORDA COUNT - UK

- UK = 77*2 + 53*2 + 63*2 + 3*17
  = 437

Points	77	53	63	17
3	N	F	A	UK
2	UK	UK	UK	C2
1	C3	C3	C3	C3
0	C4	C4	C4	C4

FIFA Members' preferences

PROBLEM SET 3 - IV - BORDA COUNT - NIGERIA

- UK = 77*2 + 53*2 + 63*2 + 17*3 = 437
- Nigeria= 77*3 + 53*1 + 63*1 + 17*2 = 381

Points	77	53	63	17
3	N	F	A	UK
2	UK	UK	UK	N
1	N	N	N	C3
0	C4	C4	C4	C4

FIFA Members' preferences

PROBLEM SET 3 - IV - BORDA COUNT - FRANCE

- UK = 77*2 + 53*2 + 63*2 + 17*3 = 437
- Nigeria= 77*3 + 53*1 + 63*1 + 17*2 = 381
- France = 77*1 + 53*3 + 63*1 + 17*2 = 333

Points	77	53	63	17
3	N	F	A	UK
2	UK	UK	UK	F
1	F	C3	F	C3
0	C4	C4	C4	C4

FIFA Members' preferences

PROBLEM SET 3 - IV - BORDA COUNT - ARGENTINA

Points	77	53	63	17
3	N	F	A	UK
2	UK	UK	UK	A
1	A	A	C3	C3
0	C4	C4	C4	C4

FIFA Members' preferences

- UK = 77*2 + 53*2 + 63*2 + 17*3 = 437
- Nigeria= 77*3 + 53*1 + 63*1 + 17*2 = 381
- France = 77*1 + 53*3 + 63*1 + 17*2 = 333
- Argentina = 77*1 + 53*1 + 63*3 + 17*2 = 283

Under the *Borda count method*, The UK wins.

PROBLEM SET 3 IV - CONDORCET METHOD? UK VS NIGERIA

Points	77	53	63	17
3	N	F	A	UK
2	UK	UK	UK	C2
1	C3	C3	C3	C3
0	C4	C4	C4	C4

FIFA Members' preferences

PROBLEM SET 3 - IV - CONDORCET METHOD? UK VS NIGERIA

- UK = 53 + 63 + 17 = 133
- Nigeria = 77

Points	77	53	63	17
3	N	F	A	UK
2	UK	UK	UK	N
1	C3	N	N	C3
0	C4	C4	C4	C4

FIFA Members' preferences

PROBLEM SET 3 - IV - CONDORCET METHOD? UK VS FRANCE

- UK = 77 + 63 + 17 = 157
- France = 53

Points	77	53	63	17
3	N	F	A	UK
2	UK	UK	UK	F
1	F	C3	F	C3
0	C4	C4	C4	C4

FIFA Members' preferences

PROBLEM SET - 3 IV - CONDORCET METHOD? UK VS ARGENTINA

- UK = 77 + 53 + 17 = 147
- Argentina = 63

Points	77	53	63	17
3	N	F	A	UK
2	UK	UK	UK	A
1	A	A	A	C3
0	C4	C4	C4	C4

FIFA Members' preferences

Under the *Condorcet method*, The UK wins.

PROBLEM SET 3 - V

v) **If in addition to the above** we know that all the members who have as the first choice France, they prefer Argentina to Nigeria, which voting rule would assign the world cup to Argentina?

PROBLEM SET 3 - V - CONDORCET METHOD?

Points	77	53	63	17
3	N	F	A	UK
2	UK	UK	UK	C2
1	C3	**A**	C3	C3
0	C4	**N**	C4	C4

FIFA Members’ preferences

PROBLEM SET 3 - V - MAJORITY

We can rule out that the Argentina will win under plurality run-off based on the result of the previous question.

Nigeria wins under plurality.

We can also rule out that the Argentina will win under sequential plurality run-off based on the result.

France wins under sequential plurality run-off.

PROBLEM SET 3 - V - CONDORCET METHOD?

	1st Round	2nd Round	3er Round
Nigeria	77	81	89
France	53	65	121
Argentina	63	64	
UK	17		

FIFA members' preferences

PROBLEM SET 3 - V - CONDORCET METHOD?

- Argentina = 63
- UK = 77+53+17 = 147

Points	77	53	63	17
3	N	F	A	UK
2	UK	UK	UK	C2
1	C3	A	C3	C3
0	C4	N	C4	C4

FIFA Members' preferences

Under the Condorcet method, the UK wins

PROBLEM SET 3 - V - BORDA METHOD?

- Argentina = 77*1 + 53*1 + 63*3 + 17*2 = 353

Points	77	53	63	17
3	N	F	A	UK
2	UK	UK	UK	A
1	A	A	C3	C3
0	C4	N	C4	C4

FIFA Members' preferences

PROBLEM SET 3 - V - BORDA METHOD?

- Argentina = 77*1 + 53*1 + 63*3 + 17*2 = 353
- UK = 77*2 + 53*2 + 63*2 + 17*3 = 437

Points	77	53	63	17
3	N	F	A	UK
2	UK	UK	UK	A
1	A	A	C3	C3
0	C4	N	C4	C4

FIFA Members' preferences

Under *Borda method*, The UK wins.

PROBLEM SET 3 - V - TWO-ROUND RUNOFF?

	1st Round	2nd Round	3er Round
Nigeria	77	81	89
France	53	65	121
Argentina	63	64	
UK	17		

FIFA members' votes

PROBLEM SET 3 - V - TWO-ROUND RUNOFF? - FIRST ROUND

	1st Round	2nd Round	3er Round
Nigeria	77	81	89
France	53	65	121
Argentina	63	64	
UK	17		

FIFA members' votes

Argentina and **Nigeria** continue to the second round.

PROBLEM SET 3 - V - TWO-ROUND RUNOFF? - SECOND ROUND

- Argentina = 53 + 63 = 116
- Nigeria = 77 + 17 = 94

Points	77	53	63	17
3	N	F	A	UK
2	UK	UK	UK	N
1	C3	A	C3	C3
0	C4	N	C4	C4

FIFA Members' preferences

Argentina wins in the second round.

PROBLEM SET 4

6SSPP383 - Formal models of political economy: Preferences

Week 5

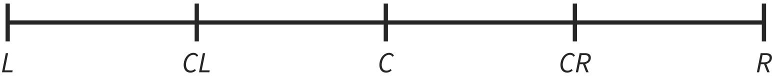
EXERCISE 1

EXERCISE 1

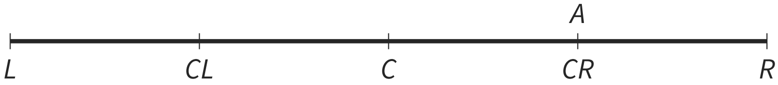
Political life in a certain polity is characterised by five discrete policy positions along a single dimension, as shown below: Left [L], Center-Left [CL], Centrist [C], Center-right [CR], and Right [R]. Voters have symmetric, **single-peaked** preferences over policy with an ideal point at one of the five positions. One fifth of the voters have an ideal point at each position. Each party announces a platform chosen from the five possible positions and is assumed to be able to commit to that platform. Given the choice between two or more parties, voters vote sincerely; when they are indifferent between two parties they fip a coin. There is no abstention.

EXERCISE 1 - A

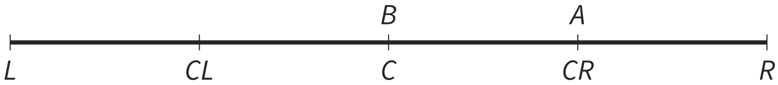
a) Find all Nash equilibria when two office-seeking parties (call them A and B) compete:



EXERCISE 1 - A

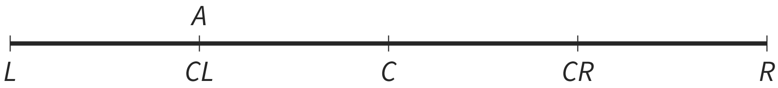


EXERCISE 1 - A



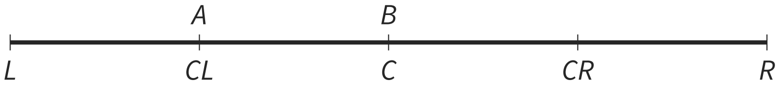
When Party A chooses Policy CR, Party B's Best Response is Policy C and wins the election.

EXERCISE 1 - A



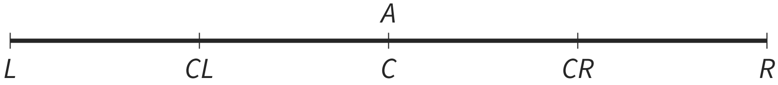
When party A chooses CL, what is party B’s Best Response?

EXERCISE 1



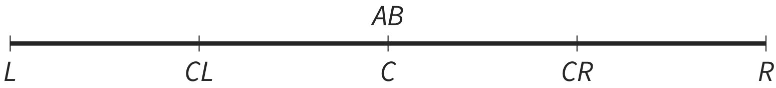
When Party A chooses Policy CL, Party B chooses Policy C. Party B wins the election.

EXERCISE 1 - A



When Party A chooses C, what is Party B’s Best Response?

EXERCISE 1 - A



When Party A chooses C, party B’s Best Response is to choose Policy C.

EXERCISE 1 - A

In summary:

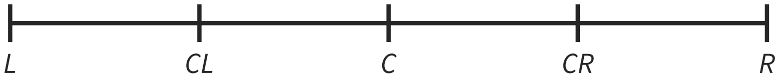
1. When party A chooses CR, Party B’s BR: C
2. When party A chooses CL, Party B’s BR: C
3. When party A chooses C, Party B’s BR: C

The unique Nash equilibrium sees both parties proposing policy C. Under a two-party system, parties converge on their policies.

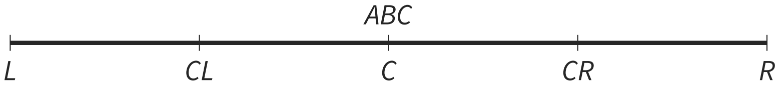
EXERCISE 1 - B

b) Find all Nash equilibria when three office-seeking parties (call them A, B, and C) compete.

EXERCISE 1 - B

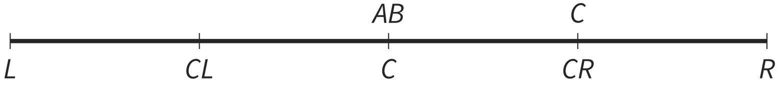


PROBLEM SET 3 - B



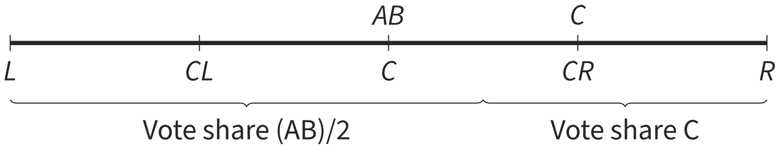
Are there any profitable deviations for any of the parties?

PROBLEM SET 3 - B



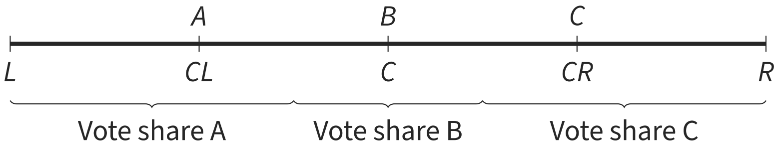
If three parties converge into one policy, one party can deviate and win the election.

PROBLEM SET 3 - B



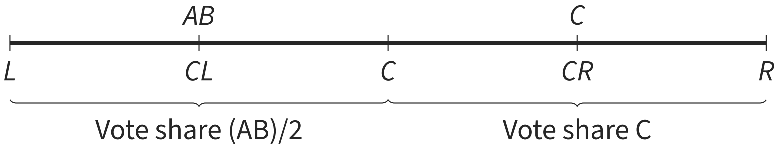
Is this a NE? Let’s see party A’s best response.

PROBLEM SET 3 - B



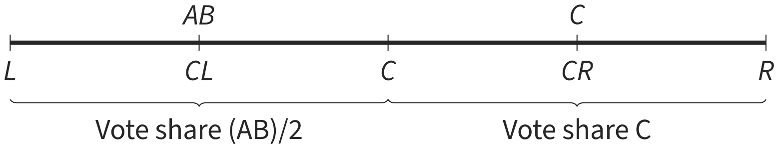
Player A best response is to move to CL, therefore {*CL*, *C*, *CR*} is a Nash Equilibrium as all three parties have the same probability to win. Player B's payoff is always a loss, regardless of whether this party deviates to CL or CR. Suppose B chooses to C; A wins for sure. If B chooses to CL, C wins.

PROBLEM SET 3 - B



Is (CL, CL, CR) a NE? Let's see what is party A's best response.

PROBLEM SET 3 - B

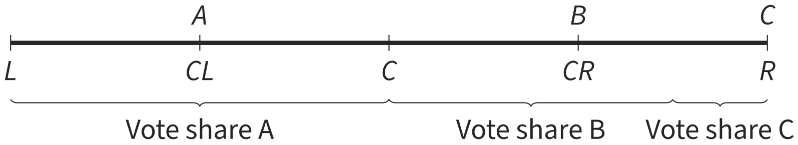


If parties A and B, set their policy platform at CL, and party C chooses CR. Party C wins the election.

- If party C chooses policy CR, and if party A chooses L, party A looses.
- If party C chooses policy CR, and if party A chooses C, party A looses.
- If party C chooses policy CR, and if party A chooses CR, party A looses.
- If party C chooses policy CR, and if party A chooses R, party A looses.

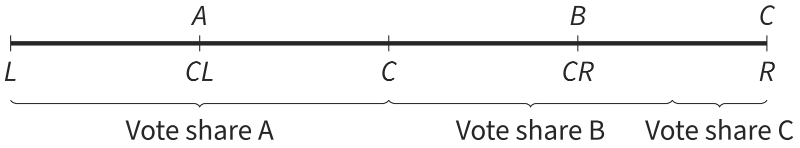
Therefore, **(CL, CL, CR) is NE**

PROBLEM SET 3 - B



Is (CL, CR, R) a NE? Let’s evaluate party B’s best response.

PROBLEM SET 3 - B



- If party A chooses CL, party C chooses R. If party B chooses CR, party B loses the election.
- If party A chooses CL, party C chooses R. If party B chooses C, party B loses the election.
- If party A chooses CL, party C chooses R. If party B chooses CL, party B loses the election.
- If party A chooses CL, party C chooses R. If party B chooses L, party B loses the election.

Thus, (CL, CR, R) is also a NE.

PROBLEM SET 3 - B

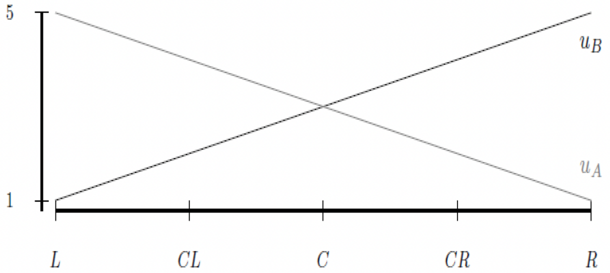
If you keep going doing this, the set of pure-strategy NE consists of:

- (CL;C,CR)
- (CL;CL;CR)
- (CL;CR;CR)
- (L,L,CR)
- (CL;R;R)
- (L;CL;CR)
- (CL;CR:R).

Note that multi-partyism leads to policy divergence. Note also that the equilibria are symmetric

EXERCISE 1 - C

c) Now suppose again that there are only two parties but that these parties care only about what policy is implemented. The vN-M utility function of party A is equal to 5 for L, 4 for CL, 3 for C, 2 for CR, 1 for R. The utility function of party B is equal to 5 for R, 4 for CR, 3 for C, 2 for CL, 1 for L; these are displayed below. Find all the Nash equilibria.



EXERCISE 1 - C

Party B (columns) / **Party A** (rows)

	L	**CL**	C	CR	R
L	5,1	4,2	3, 3	2, 4	3, 3
CL	4,2	4, 1.5	3, 3	3, 3	4, 2
C	3,3	3, 3	3, 3	3, 3	3, 3
CR	2,4	3, 3	3, 3	2, 4	2, 4
R	3, 3	4, 2	3, 3	2, 4	1, 5

EXERCISE 1 - CK

For the case of (R,L), it's a lottery:

$$E(U_A(R,L)) = \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot 5 = 3$$
$$E(U_B(R,L)) = \frac{1}{2} \cdot 5 + \frac{1}{2} \cdot 1 = 3$$

For the case of (CR,R). Party A wins:

$$E(U_A(CR,R)) = 2$$
$$E(U_B(CR,R)) = 4$$

	L	CL	C	CR	R
L	5,1	4,2	3, 3	2, 4	3, 3
CL	4,2	4, 1.5	3, 3	3, 3	4, 2
C	3,3	3, 3	3, 3	3, 3	3, 3
CR	2,4	3, 3	3, 3	2, 4	2, 4
R	3, 3	4, 2	3, 3	2, 4	1, 5

Y: Party A, X: Party B

EXERCISE 1 - C

For the case of (CL,CR). It's a lottery:

$$E[U_A(CL, CR)] = \frac{1}{2} \cdot 4 + \frac{1}{2} \cdot 2 = 3$$
$$E[U_B(CL, cR)] = \frac{1}{2} \cdot 2 + \frac{1}{2} \cdot 4 = 3$$

	L	CL	C	CR	R
L	5,1	4,2	3, 3	2, 4	3, 3
CL	4,2	4, 1.5	3, 3	3, 3	4, 2
C	3,3	3, 3	3, 3	3, 3	3, 3
CR	2,4	3, 3	3, 3	2, 4	2, 4
R	3, 3	4, 2	3, 3	2, 4	1, 5

Y: Party A, X: Party B

EXERCISE 1 - C

	L	CL	C	CR	R
L	5,1	4,2	3, 3	2, 4	3, 3
CL	4,2	4, 1.5	3, 3	3, 3	4, 2
C	3,3	3, 3	3, 3	3, 3	3, 3
CR	2,4	3, 3	3, 3	2, 4	2, 4
R	3, 3	4, 2	3, 3	2, 4	1, 5

Y: Party A, X: Party B

The set of policies that constitute a Nash equilibria consists of (C;C), (CL;C), (C;CR), and (CL; CR).

EXERCISE 1 - C

For (CL, C), party B wins, **and implements C**, so the utility of each party is equal to:

$$U_A(p_B) = 3$$

$$U_B(p_B) = 3$$

For (CL, CR), there is draw, so the expected utility of each party is equal to:

$$E(U_A) = \frac{1}{2} \cdot 4 + \frac{1}{2} \cdot 2 = 3$$

$$E(U_B) = \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot 4 = 3$$

EXERCISE 1 - C

Finally (C, C), there is a draw where both commit to implement C.

$$E(U_A) = \frac{1}{2} \cdot 3 + \frac{1}{2} \cdot 3 = 3$$
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Parties in this case we assumed are risk adverse. They prefer a policy closer to their most preferred policy with certainty over a lottery between their most desirable and less desirable policies.

EXERCISE 1 - C

Note that because of risk-neutrality we can get policy-divergent equilibria, since parties will be indifferent between a sure outcome of 3 if they win the election outright with a centrist platform, or an expected outcome of 3 if they have an 1/2 chance of winning with a platform closer to their ideal point (CL or CR). Note that these equilibria are not strict in the sense that any deviation from the equilibrium strategy does not necessarily leave one strictly worse off.

EXERCISE 1 - D

d) What happens in the situation from part (c) if we assume that parties cannot commit to their platforms?

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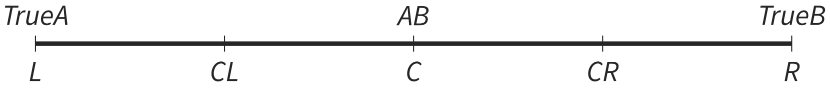
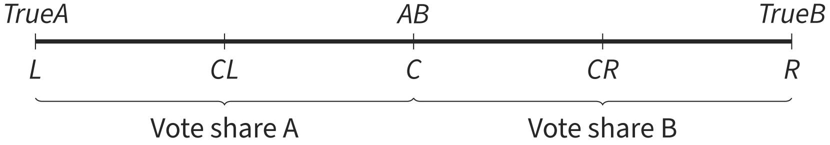


If parties cannot commit to their platforms, **then voters can anticipate that the winning party will implement its ideal policy at either of the two extremes L and R**.

EXERCISE 1 - D

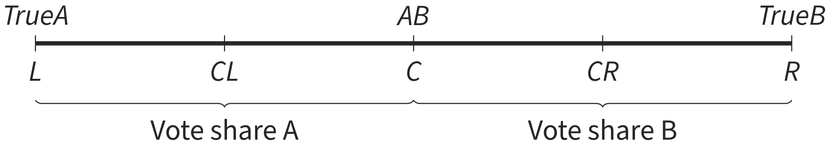
Hence, all voters located at L and CL and half of the centrist voters located at C will vote for party A, and all voters located at R and CR and half of the centrist voters located at C will vote for party B.



Hence, all moderate/non-extremist policy platforms and promises at the pre- electoral stage become non-credible ’cheap talk’ and all such policy configurations become degenerate (trivial) Nash equilibria.

EXERCISE 1 - D

If you assume that policy platforms are non-credible only in the eyes of other parties, while voters continue to believe in parties' ability to commit to their promises, then this extension becomes equivalent to office-seeking Downsian competition with the only NE occurring at (C;C).



EXERCISE 1 - E

e) Now (again assuming policy commitment) imagine that party A is policy-orientated (it has the preferences described in part (c)) while party B is office- seeking. Find the Nash equilibria. (optional and a bit hard).

EXERCISE 1 - C

$$U_A = \begin{cases} 5, & \text{if p = L} \\ 4, & \text{if p = CL} \\ 3, & \text{if p = C} \\ 2, & \text{if p = CR} \\ 1, & \text{if p = R} \end{cases}$$

$$U_B = \begin{cases} 1, & \text{if Party B wins} \\ 0, & \text{IF Tie} \\ -1, & \text{if Party A wins} \end{cases}$$

EXERCISE 1 - E

Party B

Office-motivated

Party A

Policy-orientated

	L	CL	C	CR	R
L	5,0	4, 1	3, 1	2, 1	3, 0
CL	4, -1	4, 0	3, 1	3, 0	4, -1
C	3, -1	3, -1	3, 0	3, -1	3, -1
CR	2, -1	3, 0	3, 1	2, 0	2, -1
R	3, 0	4, 1	3, 1	2, 1	1, 0

EXERCISE 1 - E

	L	CL	C	CR	R
L	5, 0	4, 1	3, 1	2, 1	3, 0
CL	4, -1	4, 0	3, 1	3, 0	4, -1
C	3, -1	3, -1	3, 0	3, -1	3, -1
CR	2, -1	3, 0	3, 1	2, 0	2, -1
R	3, 0	4, 1	3, 1	2, 1	1, 0

Y axis: Policy-orientated - X axis: Office-motivated

So the set of NE consists of (L;C), (CL:C), (C;C), (CR;C), (R;C), (L:CL) and (R;CL).

EXERCISE 1 - E

So the set of NE consists of (L;C), (CL:C), (C;C), (CR;C), (R;C), (L:CL) and (R;CL).

In other words, the office-seeking party will either locate at the center and guarantee itself the highest chance of winning, or it may be pulled to a winning position (CL) closer to the policy-seeking party's ideal point (L) if an only if the latter has adopted a losing extremist platform (L or R).

EXERCISE 1 - F

f) Try to do it on your own time. It’s just simply changing the payoffs.

PROBLEM SET 5

6SSPP383 - Formal models of political economy: Policy platforms

Week 7

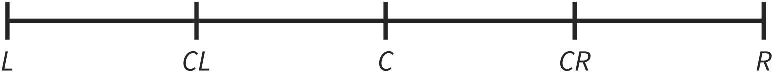
EXERCISE 1

EXERCISE 1

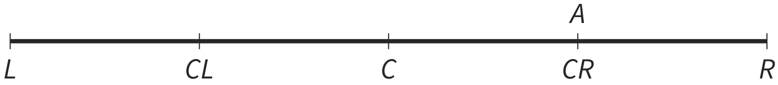
Political life in a certain polity is characterised by five discrete policy positions along a single dimension, as shown below: Left [L], Center-Left [CL], Centrist [C], Center-right [CR], and Right [R]. Voters have symmetric, **single-peaked** preferences over policy with an ideal point at one of the five positions. One fifth of the voters have an ideal point at each position. Each party announces a platform chosen from the five possible positions and is assumed to be able to commit to that platform. Given the choice between two or more parties, voters vote sincerely; when they are indifferent between two parties they fip a coin. There is no abstention.

EXERCISE 1 - A

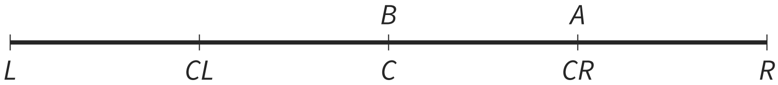
a) Find all Nash equilibria when two office-seeking parties (call them A and B) compete:



EXERCISE 1 - A

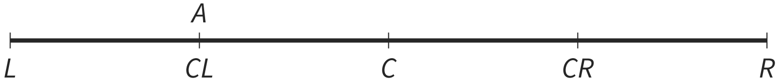


EXERCISE 1 - A



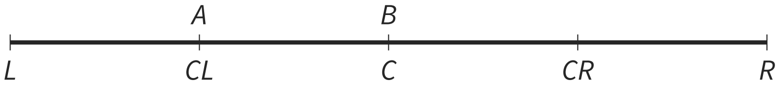
When Party A chooses Policy CR, Party B's Best Response is Policy C and wins the election.

EXERCISE 1 - A



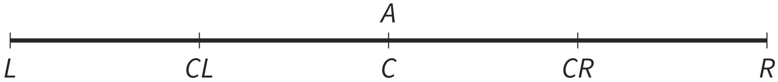
When party A chooses CL, what is party B’s Best Response?

EXERCISE 1



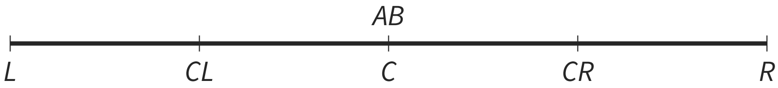
When Party A chooses Policy CL, Party B chooses Policy C. Party B wins the election.

EXERCISE 1 - A



When Party A chooses C, what is Party B's Best Response?

EXERCISE 1 - A



When Party A chooses C, party B’s Best Response is to choose Policy C.

EXERCISE 1 - A

In summary:

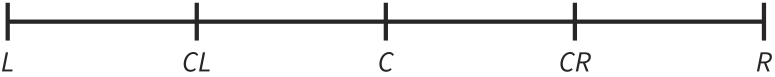
1. When party A chooses CR, Party B’s BR: C
2. When party A chooses CL, Party B’s BR: C
3. When party A chooses C, Party B’s BR: C

The unique Nash equilibrium sees both parties proposing policy C. Under a two-party system, parties converge on their policies.

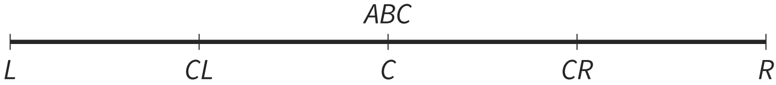
EXERCISE 1 - B

b) Find all Nash equilibria when three office-seeking parties (call them A, B, and C) compete.

EXERCISE 1 - B

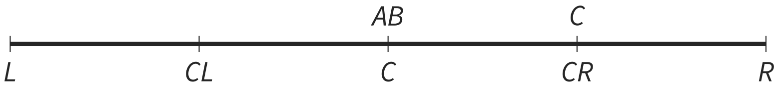


PROBLEM SET 3 - B



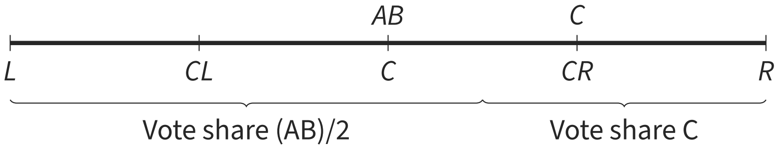
Are there any profitable deviations for any of the parties?

PROBLEM SET 3 - B



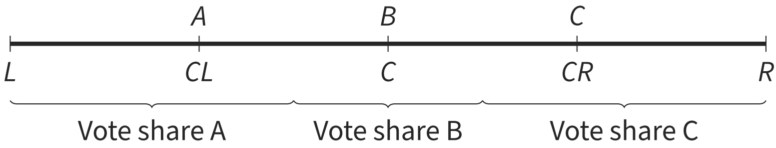
If three parties converge into one policy, one party can deviate and win the election.

PROBLEM SET 3 - B



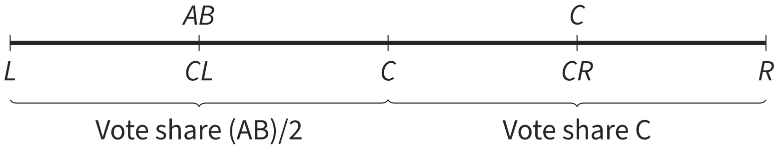
Is this a NE? Let’s see party A’s best response.

PROBLEM SET 3 - B



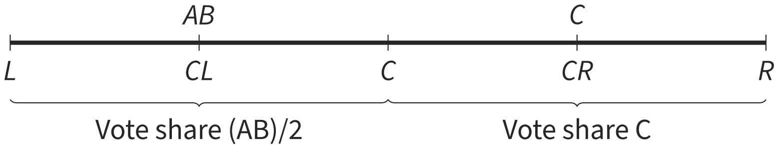
Player A best response is to move to CL, therefore {*CL*, *C*, *CR*} is a Nash Equilibrium as all three parties have the same probability to win. Player B's payoff is always a loss, regardless of whether this party deviates to CL or CR. Suppose B chooses to C; A wins for sure. If B chooses to CL, C wins.

PROBLEM SET 3 - B



Is (CL, CL, CR) a NE? Let’s see what is party A’s best response.

PROBLEM SET 3 - B

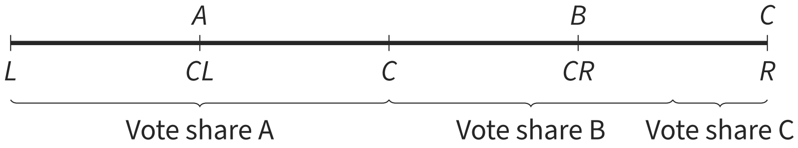


If parties A and B, set their policy platform at CL, and party C chooses CR. Party C wins the election.

- If party C chooses policy CR, and if party A chooses L, party A looses.
- If party C chooses policy CR, and if party A chooses C, party A looses.
- If party C chooses policy CR, and if party A chooses CR, party A looses.
- If party C chooses policy CR, and if party A chooses R, party A looses.

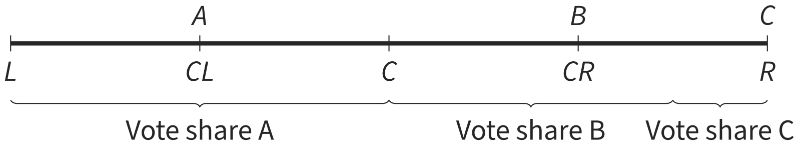
Therefore, **(CL, CL, CR) is NE**

PROBLEM SET 3 - B



Is (CL, CR, R) a NE? Let’s evaluate party B’s best response.

PROBLEM SET 3 - B



- If party A chooses CL, party C chooses R. If party B chooses CR, party B loses the election.
- If party A chooses CL, party C chooses R. If party B chooses C, party B loses the election.
- If party A chooses CL, party C chooses R. If party B chooses CL, party B loses the election.
- If party A chooses CL, party C chooses R. If party B chooses L, party B loses the election.

Thus, (CL, CR, R) is also a NE.

PROBLEM SET 3 - B

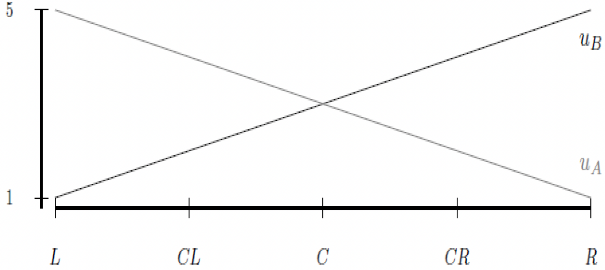
If you keep going doing this, the set of pure-strategy NE consists of:

- (CL;C,CR)
- (CL;CL;CR)
- (CL;CR;CR)
- (L,L,CR)
- (CL;R;R)
- (L;CL;CR)
- (CL;CR:R).

Note that multi-partyism leads to policy divergence. Note also that the equilibria are symmetric

EXERCISE 1 - C

c) Now suppose again that there are only two parties but that these parties care only about what policy is implemented. The vN-M utility function of party A is equal to 5 for L, 4 for CL, 3 for C, 2 for CR, 1 for R. The utility function of party B is equal to 5 for R, 4 for CR, 3 for C, 2 for CL, 1 for L; these are displayed below. Find all the Nash equilibria.



EXERCISE 1 - C

		Party B				
		L	**CL**	C	CR	R
Party A	L	5,1	4,2	3, 3	2, 4	3, 3
	CL	4,2	4, 1.5	3, 3	3, 3	4, 2
	C	3,3	3, 3	3, 3	3, 3	3, 3
	CR	2,4	3, 3	3, 3	2, 4	2, 4
	R	3, 3	4, 2	3, 3	2, 4	1, 5

EXERCISE 1 - CK

For the case of (R,L), it's a lottery:

$$E(U_A(R,L)) = \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot 5 = 3$$
$$E(U_B(R,L)) = \frac{1}{2} \cdot 5 + \frac{1}{2} \cdot 1 = 3$$

For the case of (CR,R). Party A wins:

$$E(U_A(CR,R)) = 2$$
$$E(U_B(CR,R)) = 4$$

	L	CL	C	CR	R
L	5,1	4,2	3, 3	2, 4	3, 3
CL	4,2	4, 1.5	3, 3	3, 3	4, 2
C	3,3	3, 3	3, 3	3, 3	3, 3
CR	2,4	3, 3	3, 3	2, 4	2, 4
R	3, 3	4, 2	3, 3	2, 4	1, 5

Y: Party A, X: Party B

EXERCISE 1 - C

For the case of (CL,CR). It's a lottery:

$$E[U_A(CL, CR)] = \frac{1}{2} \cdot 4 + \frac{1}{2} \cdot 2 = 3$$
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	L	CL	C	CR	R
L	5,1	4,2	3, 3	2, 4	3, 3
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EXERCISE 1 - C

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The set of policies that constitute a Nash equilibria consists of (C;C), (CL;C), (C;CR), and (CL; CR).

EXERCISE 1 - C

For (CL, C), party B wins, **and implements C**, so the utility of each party is equal to:

$$U_A(p_B) = 3$$
$$U_B(p_B) = 3$$

For (CL, CR), there is draw, so the expected utility of each party is equal to:

$$E(U_A) = \frac{1}{2} \cdot 4 + \frac{1}{2} \cdot 2 = 3$$
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EXERCISE 1 - C

Finally (C, C), there is a draw where both commit to implement C.

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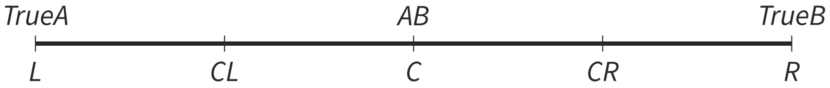

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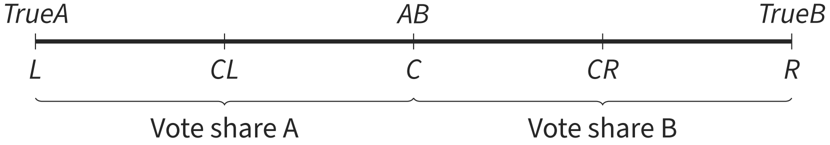
Hence, all voters located at L and CL and half of the centrist voters located at C will vote for party A, and all voters located at R and CR and half of the centrist voters located at C will vote for party B.



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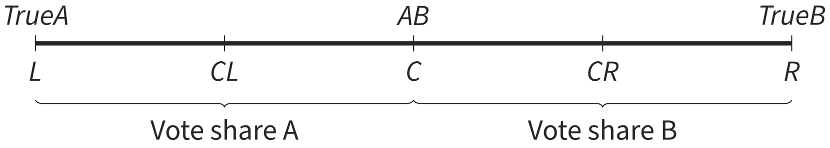

EXERCISE 1 - E

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EXERCISE 1 - C

$$U_A = \begin{cases} 5, & \text{if } p = L \\ 4, & \text{if } p = CL \\ 3, & \text{if } p = C \\ 2, & \text{if } p = CR \\ 1, & \text{if } p = R \end{cases}$$

$$U_B = \begin{cases} 1, & \text{if Party B wins} \\ 0, & \text{IF Tie} \\ -1, & \text{if Party A wins} \end{cases}$$

EXERCISE 1 - E

Party B

Office-motivated

Party A

Policy-orientated

	L	CL	C	CR	R
L	5,0	4, 1	3, 1	2, 1	3, 0
CL	4, -1	4, 0	3, 1	3, 0	4, -1
C	3, -1	3, -1	3, 0	3, -1	3, -1
CR	2, -1	3, 0	3, 1	2, 0	2, -1
R	3, 0	4, 1	3, 1	2, 1	1, 0

EXERCISE 1 - E

	L	CL	C	CR	R
L	5, 0	4, 1	3, 1	2, 1	3, 0
CL	4, -1	4, 0	3, 1	3, 0	4, -1
C	3, -1	3, -1	3, 0	3, -1	3, -1
CR	2, -1	3, 0	3, 1	2, 0	2, -1
R	3, 0	4, 1	3, 1	2, 1	1, 0

Y axis: Policy-orientated - X axis: Office-motivated

So the set of NE consists of (L;C), (CL:C), (C;C), (CR;C), (R;C), (L:CL) and (R;CL).

EXERCISE 1 - E

So the set of NE consists of (L;C), (CL:C), (C;C), (CR;C), (R;C), (L:CL) and (R;CL).

In other words, the office-seeking party will either locate at the center and guarantee itself the highest chance of winning, or it may be pulled to a winning position (CL) closer to the policy-seeking party’s ideal point (L) if an only if the latter has adopted a losing extremist platform (L or R).

EXERCISE 1 - F

f) Try to do it on your own time. It’s just simply changing the payoffs.

PROBLEM SET 6

6SSPP383 - Formal models of political economy: Redistribution

EXERCISE 1 - REDISTRIBUTIVE POLITICS IN THE MELTZER-RICHARD WORLD

EXERCISE 1

Consider a society divided into two groups, the rich (r) and the poor (p) with respective utility functions u_r and u_p such that:

$$u_r(t, y) = y(1 - t) - \frac{1}{2} y^2 \quad (1)$$

$$u_p(t, y) = t\,y \quad (2)$$

where y is income generated (at a cost) by the rich and t is the tax rate. Most members of the society are poor.

SET UP

Let’s think about the set up:

- Rich and poor have different utilities functions
- Taxes are levied from the rich against their income
- The tax rate t is a constant fraction of earned income, but declining fraction of disposable income
- The only source of income of the poor are lump-sump redistribution payments (which are also taxed)

EXERCISE 1 - A

a) Derive the income generated by the rich as a function of the tax rate.

EXERCISE 1 - A

In order to derive the income generated by the rich as a function of the tax rate, it suffices to take the first derivative of the utility function with respect to income (in light of the concavity of the utility function with respect to y), i.e.,

$$u_r(t, y) = y(1 - t) - \frac{1}{2} y^2 \tag{3}$$

$$\frac{\partial u_r(t, y)}{\partial y} = 0 \tag{4}$$

$$y - yt - \frac{1}{2} y^2 = 0 \tag{5}$$

The rich are optimising income(labour vs leisure) as a function of the tax rate.

EXERCISE 1 - A

$$y - yt - \frac{1}{2} y^2 = 0 \tag{6}$$

$$\frac{\partial y}{\partial y} - \frac{\partial yt}{\partial y} - \frac{1}{2} \frac{\partial y^2}{\partial y} = 0 \tag{7}$$

$$1 - t - \frac{2}{2} y^{2-1} = 0 \tag{8}$$

$$(1 - t) - y^* = 0 \tag{9}$$

$$y^* = 1 - t \tag{10}$$

This specification captures the disincentive labor supply effects of taxation.

EXERCISE 1 - B

b) What is the tax rate that would be set by the poor if **y was constant**?

EXERCISE 1 - B

$$u_p(t, y) = t\,y \tag{11}$$

EXERCISE 1 - B

$$u_p(t, y) = t\,y \tag{12}$$

Linear utility, so no diminishing marginal utility.

EXERCISE 1 - B

$$u_p(t, y) = t\,y \tag{13}$$

Since the poor do not face any disincentive effects from taxation and their income is wholly dependent on re-distributive transfers, they would prefer a maximum tax rate of 1.

EXERCISE 1 - C

c) Suppose that the tax rate is set by politicians competing for office in a democracy in which all citizens vote. Find the (Nash) equilibrium t and y:

EXERCISE 1 - C

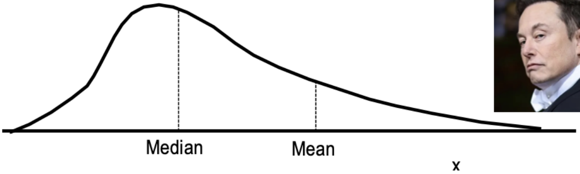
Since the median voter is poor, then the equilibrium tax rate produced by electoral competition will be the one that maximises the income of the poor, i.e.,

$$u_p(t, y) = t\,y \tag{14}$$

- Voters disagree over their desire fiscal policy **Rich** (t = 0) and the **Poor** (t =1)
- Candidates must decide which voters to please to enhance their chances of winning
- The income distribution is no denegerate (skewed)

EXERCISE 1 - C

The median voter is poor



So everyone with $y^i < y^p$ prefers a larger government, so more than half would be for the party that selects t^p. Then, there is a unique equilibrium

$$t_A = t_B = t_m$$

EXERCISE 1 - C

Since the median voter is poor, then the equilibrium tax rate produced by electoral competition will be the one that maximises the income of the poor, i.e.,

$$u_p = m(t, y) = t\,y \tag{15}$$

All the income for the poor comes from transfers:

$$t^* = \arg\max_t u_p = m(t, y^*(t)) \tag{16}$$

$$t^* = \arg\max_t t\,y^*(t) \tag{17}$$

EXERCISE 1 - C

$$t^* = \arg\max_t t\, y^*(t) \tag{18}$$

We know from a) that: $y^* = 1 - t$

$$t^* = \arg\max_t t(1 - t) \tag{19}$$

We need to get the F.O.C to that maximises the median voter's utility:

$$t^* = \arg\max_t t - t^2 \tag{20}$$

EXERCISE 1 - C

We need to get the F.O.C to that maximises the median voter's utility:

$$t^* = \arg\max_t t - t^2 \tag{21}$$

$$\frac{\partial t}{\partial t} - \frac{\partial t^2}{\partial t} = 0 \tag{22}$$

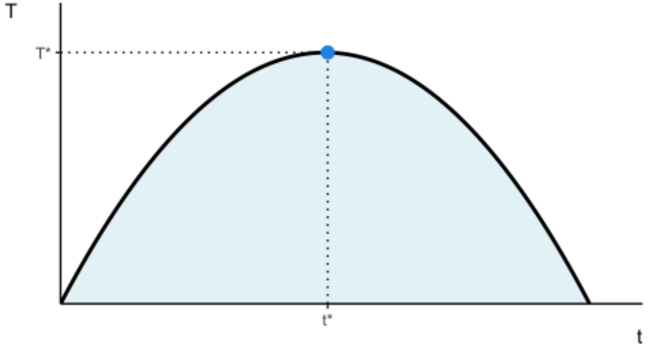
$$1 - 2t^{2-1} = 0 \tag{23}$$

$$1 - 2t = 0 \tag{24}$$

$$t^* = \frac{1}{2} \tag{25}$$

EXERCISE 1 - C

This quadratic function is maximised at $t^* = \frac{1}{2}$, where the income of the rich becomes $y^* = 1 - t \rightarrow y^* = 1 - \frac{1}{2}$. Note that this is the tax rate that maximises tax revenues from the rich (peak of the Laffer curve).



A higher tax leads to a lower tax base.

EXERCISE 1 - C

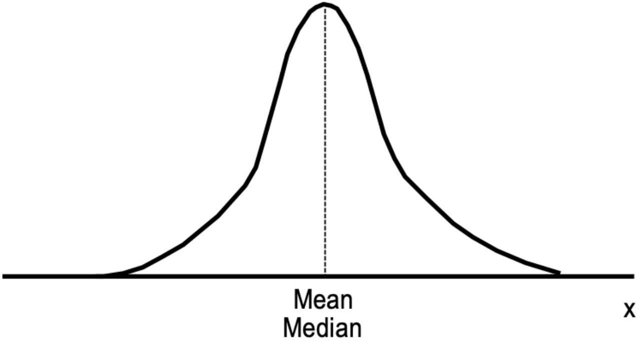
Thus each voter trades off the marginal re-distributive benefit (or cost) of taxation against its deadweight loss.

This is a sensible prediction: Voters rationally anticipate the discentive effects of taxation on the labour-leisure choices of their fellow citizens and take into account when voting.

Increases in the tax rate has two effects: "Each pound earned income raises more revenue, but earned income declines; the rich choose more leisure.".

EXERCISE 1 - C

So, the more unequal society, where the rich most of the wealth will predict a higher tax rate.



In the case above, the model will predict a lower tax rate (compared to the skewed distribution).

EXERCISE 1 - D

d) Now suppose that the rich have an exogenously-determined asset **z** that is subject to the same tax rate t. (For example, **z** could be land wealth, where all land is owned by the rich and taxed at the same rate as income).

Rewrite the utility functions to reflect this situation and express the new equilibrium t and y.

EXERCISE 1 - D

$$u_r(t, y) = y(1 - t) - \frac{1}{2} y^2 \tag{26}$$

$$u_p(t, y) = t\, y \tag{27}$$

EXERCISE 1 - D

Utility functions become as follows:

$$u_r(t, y, z) = (y + z)(1 - t) - \frac{1}{2} y^2 \tag{28}$$

$$u_p(t, y, z) = t(y + z) \tag{29}$$

EXERCISE 1 - D

Following the same maximisation approach as above, we get that:

$$t^* = \arg\max_t u_p(t,\ y^*(t), z) \tag{30}$$

$$t^* = \arg\max_t u_p(t,\ y^*(t), z) = t(y^*(t) + z) \tag{31}$$

We need to derive the income generated by the rich as the function of the tax rate:

$$u_r(t,\ y, z) = (y + z)(1 - t) - \frac{1}{2}y^2 \tag{32}$$

EXERCISE 1 - D

F.O.C wrt to Y for the **Rich**:

$$u_r(t,\ y, z) = (y + z)(1 - t) - \frac{1}{2}y^2 \tag{33}$$

$$\frac{\partial u_r(t,\ y, z)}{\partial y} = y - yt + z - zt - \frac{1}{2}y^2 = 0 \tag{34}$$

$$\frac{\partial u_r(t,\ y, z)}{\partial y} = \frac{\partial y}{\partial y} - \frac{\partial yt}{\partial y} + \frac{\partial z}{\partial y} - \frac{\partial zt}{\partial y} - \frac{1}{2}\frac{\partial y^2}{\partial y} \tag{35}$$

$$\frac{\partial u_r(t,\ y, z)}{\partial y} = 1 - t + 0 - 0 - y^* = 0 \tag{36}$$

$$1 - t - y^* = 0 \Rightarrow y^* = 1 - t \tag{37}$$

EXERCISE 1 - D

F.O.C wrt to Y for the **Rich**:

$$u_r(t, y, z) = (y + z)(1 - t) - \frac{1}{2} y^2 \tag{38}$$

$$\frac{\partial u_r(t, y, z)}{\partial y} = y - yt + z - zt - \frac{1}{2} y^2 = 0 \tag{39}$$

$$\frac{\partial u_r(t, y, z)}{\partial y} = \frac{\partial y}{\partial y} - \frac{\partial yt}{\partial y} + \frac{\partial z}{\partial y} - \frac{\partial zt}{\partial y} - \frac{1}{2}\frac{\partial y^2}{\partial y} \tag{40}$$

$$\frac{\partial u_r(t, y, z)}{\partial y} = 1 - t + 0 - 0 - y^* = 0 \tag{41}$$

$$1 - t - y^* = 0 \Rightarrow y^*(t) = 1 - t \tag{42}$$

EXERCISE 1 - D

Let’s plug the revenue from taxing the rich into the poor’s utility function:

$$y^* = 1 - t \tag{43}$$

$$t^* = \arg\max_t u_p(t,\ y^*(t), z) = t(y^*(t) + z) \tag{44}$$

$$t^* = \arg\max_t u_p(t,\ y^*(t), z) = t((\underbrace{1 - t}_{y^*(t)} + z) \tag{45}$$

$$t^* = \arg\max_t u_p(t,\ y^*(t), z) = t - t^2 + tz \tag{46}$$

EXERCISE 1 - D

Continuing with the derivation:

$$\frac{\partial u_p(t, y^*(t), z)}{\partial t} = \frac{\partial t}{\partial t} - \frac{\partial t^2}{\partial t} + \frac{\partial tz}{\partial t} = 0 \quad (47)$$

$$1 - 2t^{2-1} + z = 0 \quad (48)$$

$$1 - 2t + z = 0 \quad (49)$$

$$2t = 1 + z \quad (50)$$

$$t^* = \frac{1 + z}{2} \quad (51)$$

EXERCISE 1 - D

The higher the immobile (perfectly inelastic) asset, the higher the equilibrium rate of taxation. The poor will have stronger incentive to expropriate the rich, when the main source of the latter's wealth is a specific and easily taxable asset such as land. Labour supply consideration become less important.

$$\frac{1+z}{2} > \frac{1}{2} \tag{52}$$

$$\frac{1}{2} + \frac{z}{2} > \frac{1}{2} \tag{53}$$

The poor outweight the renueve loss from income tax by the gain of taxing a higher rate the asset.

EXERCISE 1 - D

The poor more than out-weight the revenue loss from income tax by the gain of taxing the taxable asset at a higher rate.

$$\text{Taxable asset tax renueve} = (\frac{1+z}{2)}z \quad (55)$$

$$\text{Income tax revenue} = (1 - (\frac{1+z}{2})) \, y \quad (56)$$

EXERCISE 1 - E

e) Briefly discuss your results and they might related to the Meltzer-Richard model of redistribution

EXERCISE 1 - E

e) This is a simplied version of the Meltzer-Richard model.

- Excludes different levels of productivity, but different income levels capture it.
- Excludes the trade-off between labour and leisure (n and l)
- The size of the government changes also if there are changes in the relative income or relative productivity
- In this version of the model, the rich do not benefit from taxation (no lump sum transfers)
- A stronger incentive to tax assets that do not generate a deadweight loss

EXERCISE 1 - E

Their original model, it's more explicit of the trade-off between a higher tax rate and the deadweight loss that its generates

$$\tau^i = \frac{e^i - e}{L_\tau(\tau^i)} \tag{57}$$

- $e^i - e$ marginal benefit of a high tax rate, e is equal to the average productivity
- $L_\tau(\tau^i)$ marginal benefit of a smaller tax base, $L_\tau(\tau^i) < 0$

PROBLEM SET 7

6SSPP383 - Formal models of political economy: Social conflict

SEMINAR "SOCIAL CONFLICT"

SOCIAL CONFLICT

There are N individuals in one group. Each individual i in any of the two groups can exert effort x_i at cost $c(x_i) = x_i$ to increase the chance that their group obtain a private good of value V, which will then be redistributed equally across members (every one gets V/N). Denote by $\pi \in [0, 1](1 - \pi)$ the probability that group A wins the private good. Then, define X as the total effort (i.e. the sum) of group members and $\pi = \frac{X}{X+1}$ as the probability of getting the prize. Here we are interested in *individual levels of effort*.

1) Write down the utility function for one group member assuming that every member does the same effort ($X = x_i \times N$)

QUESTION 1

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- Individual cost $c(x_i) = x_i$
- Everyone gets an equal part of the private good $\frac{V}{N}$ with probability $\pi = \frac{X}{X+1}$

QUESTION 1

- Individual cost $c(x_i) = x_i$
- Everyone gets an equal part of the private good $\frac{V}{N}$ with probability $\pi = \frac{X}{X+1}$

$$U(x_i) = \frac{V}{N} \cdot \frac{X}{X+1} - c(x_i) \tag{1}$$

QUESTION 1

- Individual cost $c(x_i) = x_i$
- Everyone gets an equal part of the private good $\frac{V}{N}$ with probability $\pi = \frac{X}{X+1}$
- We know that $X = x_i \times N$, we can replace in the utility function

$$U(x_i) = \frac{V}{N} \cdot \frac{X}{X+1} - c(x_i) \tag{2}$$

QUESTION 1

We know that $X = x_i \times N$, we can replace this into equation 41

$$U(x_i) = \frac{V}{N} \cdot \frac{X}{X+1} - x_i \tag{3}$$

$$U(x_i) = \frac{V}{N} \cdot \frac{x_i N}{x_i N + 1} - x_i \tag{4}$$

$$U(x_i) = \frac{V}{\cancel{N}} \cdot \frac{x_i \cancel{N}}{x_i N + 1} - x_i \tag{5}$$

$$U(x_i) = V \cdot \left(\frac{x_i}{x_i N + 1} \right) - x_i \tag{6}$$

QUESTION 2.

2. What is the optimal effort (optimal value of x^*) ? Does optimal effort increase or decrease when N increases? Interpret this last result

QUESTION 2

How do we yield the optimal level of effort?

QUESTION 2

We derive $U(x_i)$ wrt to x.

We capture the level at which an additional unit of effort yields no extra utility.

QUESTION 2

We derive wrt to x:

$$U(x_i) = V \cdot \left(\frac{x_i}{x_i N + 1} \right) - x_i \tag{7}$$

We need to apply the **Quotient Rule:**

$$\frac{d}{dx_i} \left(\frac{f(x)}{g(x)} \right) = \frac{\frac{d}{dx} f(x)\, g(x) - f(x) \frac{d}{dx} g(x)}{g^2(x)} \tag{8}$$

$$\frac{\partial U}{\partial x_i} = V \cdot \left(\frac{\frac{\partial x_i}{\partial x_i} (x_i N + 1) - x_i \frac{\partial (x_i N + 1)}{\partial x_i}}{(x_i N + 1)^2} \right) - \frac{\partial x_i}{x_i} = 0 \tag{9}$$

$$\frac{\partial U}{\partial x_i} = V \cdot \left(\frac{(x_i N + 1) - x_i N)}{(x_i N + 1)^2} \right) - 1 = 0 \tag{10}$$

QUESTION 2

$$V \cdot \frac{x_i N + 1 - x_i N}{(x_i N + 1)^2} - 1 = 0 \tag{11}$$

$$V \cdot \frac{1}{(x_i N + 1)^2} = 1 \tag{12}$$

$$V = (x_i N + 1)^2 \tag{13}$$

$$\sqrt{V} = (x_i N + 1) \tag{14}$$

QUESTION 2

$$(x_i N + 1) = \sqrt{V} \tag{15}$$

$$x^*_{Priv} = \frac{\sqrt{V} - 1}{N} \tag{16}$$

Clearly, the value of x^*_{Priv} decreases when N increases. Technically we call this a *comparative statics*.

QUESTION 3

3. Solve again the main model, but assume that the good is public. Is there more or less free-riding than with the private good? Explain

QUESTION 3

Initially, we used the utility function in equation 19, where we cancel out the Ns. Still, now *V* is not divided equally, and now everyone can *fully* benefit from V. We are assuming the public good is nonexcludable and nonrivalrous.

$$U(x_i) = \frac{V}{N} \cdot \frac{x_i N}{x_i N + 1} - x_i \tag{17}$$

This means that each individual can take the whole benefit of V, rather than just a fraction[1]:

$$U(x_i) = \frac{V}{\cancel{N}} \cdot \frac{x_i N}{x_i N + 1} - x_i \tag{18}$$

QUESTION 3

$$U(x_i) = V \cdot \frac{x_i N}{x_i N + 1} - x_i \tag{19}$$

$$\frac{\partial U}{\partial x_i} = V \cdot \left(\frac{N(x_i N + 1) - x_i N \cdot N}{(x_i N + 1)^2} \right) - 1 = 0 \tag{20}$$

$$\frac{\partial U}{\partial x_i} = V \cdot \left(\frac{x_i N^2 + N - x_i N^2}{(x_i N + 1)^2} \right) - 1 = 0 \tag{21}$$

$$V \cdot \frac{N}{(x_i N + 1)^2} = 1 \tag{22}$$

$$V \cdot N = (x_i N + 1)^2 \tag{23}$$

QUESTION 3

$$\sqrt{V \cdot N} = x_i N + 1 \tag{24}$$

$$\frac{\sqrt{V \cdot N} - 1}{N} = x_i \tag{25}$$

$$x^*_{\text{Public}} = \frac{\sqrt{V \cdot N} - 1}{N} \tag{26}$$

QUESTION 3

Let's compare it to the result in question 2 x^*_{Priv}:

$$x^*_{Priv} = \frac{\sqrt{V} - 1}{N} \tag{27}$$

$$x^*_{\text{Public}} = \frac{\sqrt{V \cdot N} - 1}{N} \tag{28}$$

$$\sqrt{V} < \sqrt{VN}, \; N > 2 \tag{29}$$

We can see x^*_{public} is larger now than before: this means that group members make more effort, which in turn means *less* free-riding.

QUESTION 4

4. Solve again the main model (good is private) but assume that the cost is now convex: $c(x_i) = \phi x_i$, where $\phi > 0$. ϕ can be interpreted as an *inverse productivity* parameter: The lower, the lower the effort's cost. What is the value of such that there is total free riding (that is the effort is null)?

QUESTION 4

What are looking for?

QUESTION 4

What are looking for?

- We are trying to identify the *lowest* level of effort that maximises the utility of subject *i*
- Once we know that function, we can determine when x^* (effort) becomes zero or below zero.
- When $x^* \leq 0$, then individual i will free ride.

QUESTION 4

Let's use again the model, where V is a private benefit:

$$U(x_i) = V \cdot \left(\frac{x_i}{x_i N + 1} \right) - \underbrace{x_i}_{\text{but now } \phi x_i} \tag{30}$$

$$U(x_i) = V \cdot \left(\frac{x_i}{x_i N + 1} \right) - \phi x_i \tag{31}$$

We derive wrt to x_i:

$$\frac{\partial u}{\partial x_i} = V \cdot \left(\frac{(x_i N + 1) - x_i(N)}{(x_i N + 1)^2} \right) - \frac{\partial \phi x_i}{\partial x_i} \tag{32}$$

$$V \cdot \frac{1}{(x_i N + 1)^2} - \phi = 0 \tag{33}$$

QUESTION 4

$$\frac{V}{(x_i N + 1)^2} = \phi \tag{34}$$

$$V = \phi(x_i N + 1)^2 \tag{35}$$

$$\frac{V}{\phi} = (x_i N + 1)^2 \tag{36}$$

$$\frac{\sqrt{V}}{\sqrt{\phi}} = (x_i N + 1) \tag{37}$$

QUESTION 4

$$\frac{\sqrt{V}}{\sqrt{\phi}} = (x_i N + 1) \tag{38}$$

$$\frac{\sqrt{V}}{\sqrt{\phi}} - 1 = x_i N \tag{39}$$

$$\frac{\sqrt{V}}{N\sqrt{\phi}} - \frac{1}{N} = x^* \tag{40}$$

QUESTION 4

We want to know what is the value of ϕ such that individual *i* will free ride (*fully*). This means setting $x^* = 0$

$$\frac{\sqrt{V}}{N\sqrt{\phi}} - \frac{1}{N} = 0 \tag{41}$$

$$\frac{\sqrt{V}}{N\sqrt{\phi}} = \frac{1}{N} \tag{42}$$

$$V = \phi \tag{43}$$

- No effort if $V <= \phi$.
- Effort is positive as long $V > \phi$.

PROBLEM SET 8

6SSPP383 - Formal models of political economy: Social conflict II

SEMINAR "SOCIAL CONFLICT"

SOCIAL CONFLICT

There are N individuals in one group. Each individual i in any of the two groups can exert effort x_i at cost $c(x_i) = x_i$ to increase the chance that their group obtain a private good of value V, which will then be redistributed equally across members (every one gets V/N). Denote by $\pi \in [0, 1](1 - \pi)$ the probability that group A wins the private good. Then, define X as the total effort (i.e. the sum) of group members and $\pi = \frac{X}{X+1}$ as the probability of getting the prize. Here we are interested in *individual levels of effort*.

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QUESTION 1

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$$U(x_i) = V \cdot \left(\frac{x_i}{x_i N + 1} \right) - x_i \tag{6}$$

QUESTION 2.

2. What is the optimal effort (optimal value of x^*) ? Does optimal effort increase or decrease when N increases? Interpret this last result

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Clearly, the value of x^*_{Priv} decreases when N increases. Technically we call this a *comparative statics*.

QUESTION 3

3. Solve again the main model, but assume that the good is public. Is there more or less free-riding than with the private good? Explain

QUESTION 3

Initially, we used the utility function in equation 19, where we cancel out the Ns. Still, now *V* is not divided equally, and now everyone can *fully* benefit from V. We are assuming the public good is nonexcludable and nonrivalrous.

$$U(x_i) = \frac{V}{N} \cdot \frac{x_i N}{x_i N + 1} - x_i \tag{17}$$

This means that each individual can take the whole benefit of V, rather than just a fraction[1]:

$$U(x_i) = \frac{V}{\cancel{N}} \cdot \frac{x_i N}{x_i N + 1} - x_i \tag{18}$$

QUESTION 3

$$U(x_i) = V \cdot \frac{x_i N}{x_i N + 1} - x_i \tag{19}$$

$$\frac{\partial U}{\partial x_i} = V \cdot \left(\frac{N(x_i N + 1) - x_i N \cdot N}{(x_i N + 1)^2} \right) - 1 = 0 \tag{20}$$

$$\frac{\partial U}{\partial x_i} = V \cdot \left(\frac{x_i N^2 + N - x_i N^2}{(x_i N + 1)^2} \right) - 1 = 0 \tag{21}$$

$$V \cdot \frac{N}{(x_i N + 1)^2} = 1 \tag{22}$$

$$V \cdot N = (x_i N + 1)^2 \tag{23}$$

QUESTION 3

$$\sqrt{V \cdot N} = x_i N + 1 \tag{24}$$

$$\frac{\sqrt{V \cdot N} - 1}{N} = x_i \tag{25}$$

$$x^*_{\text{Public}} = \frac{\sqrt{V \cdot N} - 1}{N} \tag{26}$$

QUESTION 3

Let's compare it to the result in question 2 x^*_{Priv}:

$$x^*_{Priv} = \frac{\sqrt{V} - 1}{N} \tag{27}$$

$$x^*_{\text{Public}} = \frac{\sqrt{V \cdot N} - 1}{N} \tag{28}$$

$$\sqrt{V} < \sqrt{VN}, \; N > 2 \tag{29}$$

We can see x^*_{public} is larger now than before: this means that group members make more effort, which in turn means *less* free-riding.

QUESTION 4

4. Solve again the main model (good is private) but assume that the cost is now convex: $c(x_i) = \phi x_i$, where $\phi > 0$. ϕ can be interpreted as an *inverse productivity* parameter: The lower, the lower the effort's cost. What is the value of such that there is total free riding (that is the effort is null)?

QUESTION 4

What are looking for?

QUESTION 4

What are looking for?

- We are trying to identify the *lowest* level of effort that maximises the utility of subject *i*
- Once we know that function, we can determine when x^* (effort) becomes zero or below zero.
- When $x^* \leq 0$, then individual i will free ride.

QUESTION 4

Let's use again the model, where V is a private benefit:

$$U(x_i) = V \cdot \left(\frac{x_i}{x_i N + 1} \right) - \underbrace{x_i}_{\text{but now } \phi x_i} \tag{30}$$

$$U(x_i) = V \cdot \left(\frac{x_i}{x_i N + 1} \right) - \phi x_i \tag{31}$$

We derive wrt to x_i:

$$\frac{\partial u}{\partial x_i} = V \cdot \left(\frac{(x_i N + 1) - x_i(N)}{(x_i N + 1)^2} \right) - \frac{\partial \phi x_i}{\partial x_i} \tag{32}$$

$$V \cdot \frac{1}{(x_i N + 1)^2} - \phi = 0 \tag{33}$$

QUESTION 4

$$\frac{V}{(x_i N + 1)^2} = \phi \tag{34}$$

$$V = \phi (x_i N + 1)^2 \tag{35}$$

$$\frac{V}{\phi} = (x_i N + 1)^2 \tag{36}$$

$$\frac{\sqrt{V}}{\sqrt{\phi}} = (x_i N + 1) \tag{37}$$

QUESTION 4

$$\frac{\sqrt{V}}{\sqrt{\phi}} = (x_i N + 1) \tag{38}$$

$$\frac{\sqrt{V}}{\sqrt{\phi}} - 1 = x_i N \tag{39}$$

$$\frac{\sqrt{V}}{N\sqrt{\phi}} - \frac{1}{N} = x^* \tag{40}$$

QUESTION 4

We want to know what is the value of ϕ such that individual *i* will free ride (*fully*). This means setting $x^* = 0$

$$\frac{\sqrt{V}}{N\sqrt{\phi}} - \frac{1}{N} = 0 \tag{41}$$

$$\frac{\sqrt{V}}{N\sqrt{\phi}} = \frac{1}{N} \tag{42}$$

$$V = \phi \tag{43}$$

- No effort if $V <= \phi$.
- Effort is positive as long $V > \phi$.

PROBLEM SET 9

6SSPP383 - Formal models of political economy: Lobbying and Legislative Politics

QUESTION 1

Three legislators need to decide between two policies. To interest group have vested interest in which policy is chosen. The payoff table below denote the utility each of the payoffs derive from each policy:

Actor	Policy X	Policy Y
Legislator A	0	2
Legislator B	2	0
Legislator C	5	0
Interest Group RED	5	0
Interest Group BLUE	0	4

QUESTION 1

The three legislators vote and the policy that wins a majority of the votes is implemented. Legislators favour the policy that yields most utility to them, but are willing to accept bribes by the interests groups and may switch their vote if the interest group compensates them for the lower utility they derive from their least preferred option. For instance, legislator A is originally inclined to vote for policy Y, but will vote for policy X if an interest group pays her 2 monetary units. Throughout we assume that, when a legislator is indifferent, she favours policy X.

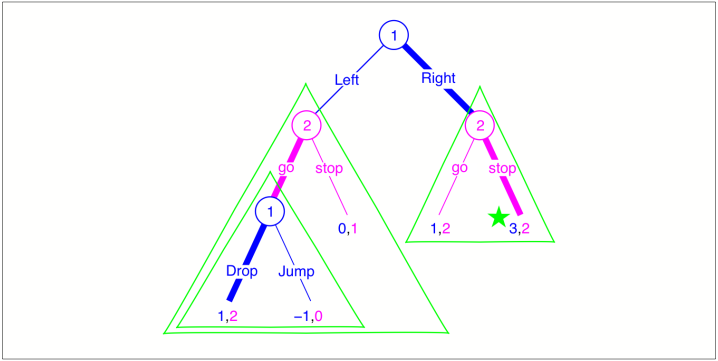
QUESTION 1

Consider the following timing of the game:

- (1) Interest Group RED has the option of bribing the legislators;
- (2) Interest Group BLUE has the option of bribing the legislators;
- (3) Legislators vote and the majoritarian option is implemented.

(i) Which policy is implemented in the subgame-perfect Nash equilibrium of the game? What are the bribes that the interest groups o¤er to the legislators?

QUESTION 1



QUESTION 1

Let's think about the set up:

- The Interest Groups (IG) will give λ_i legislators whose payoffs are the lowest.
- That is, they will give λ_i to each of those legislators (denote them by $i \in \{1, \ldots, k\}$ and she will keep $1 - \sum_{i=i,\ldots,k}(\lambda_i)$ for herself.
- The proposal is accepted by $\frac{N+1}{2}$ votes.
- Both IGs RED and BLUE can bribe more than one legislator.
- For example: in the first round, IG RED can bribe: no one, legislator A, legislator A and B, or all of them with
- Payoff are common knowledge

QUESTION 1

Let's think about the set up:

In the first round, IG RED has multiple options. It can bribe no bribes, legislator A, legislator A and B, or all of them with: No bribesABCAB, BC, ACA with 2 MUB with 2 MUC with 1 MU, and A with 3 MU

But let's remember that she will keep $1 - \sum_{i=i,\ldots,k}(\lambda_i)$ for herself.

QUESTION 1

If IG RED bribes no one:

- In the last stage of the game, BLUE tries to gain support for policy Y, but she is only willing to pay 4 monetary units (MU)
- In the second part of the game, IG BLUE will pay 3 MU to legislator B, and policy Y will be chosen.
- IG RED will anticipate this, thus, will not do this.

QUESTION 1

If IG RED bribes legislator A with 2 MU:

- In the second part of the game, IG BLUE will pay 1 MU to legislators A and to 3 MU legislator B, and policy Y will be chosen.
- IG RED will anticipate this, thus, will not bribe legislator A

QUESTION 1

IG RED tries to ensure that BLUE is not able to gain such support, but only does so if the cost are below 5 MU.

- What is the optimal strategy (less costly) to ensure policy X is chosen?

QUESTION 1

IG RED tries to ensure that BLUE is not able to gain such support but only does so if the cost is below 5 MU.

- What is the optimal (less costly) strategy to ensure policy X is chosen?
  Let's think this more broadly
  - Legislator A prefers policy Y; bribing legislator A is not optimal

Actor	Policy X	Policy Y
Legislator A	0	2
Legislator B	2	0
Legislator C	5	0
Interest Group RED	5	0
Interest Group BLUE	0	4

QUESTION 1

IG RED tries to ensure that BLUE is not able to gain such support, but only does so if the cost are below 5 MU.

- Is it optimal for IG RED to bribe legislator C?

Actor	**Policy X**	**Policy Y**
Legislator A	0	2
Legislator B	2	0
Legislator C	5	0
Interest Group RED	5	0
Interest Group BLUE	0	4

QUESTION 1

IG RED tries to ensure that BLUE is not able to gain such support, but only does so if the cost are below 5 MU.

- Is it optimal for IG RED to bribe legislator C?
  - No, because IG RED has support for X from legislator C
  - The payoff of legislator C is higher than the IG BLUE can pay (4MU)

QUESTION 1

IG RED tries to ensure that BLUE is not able to gain such support, but only does so if the cost are below 5 MU.

- Is it optimal for IG RED to bribe legislator B, and if so, for how much?

Actor	Policy X	Policy Y
Legislator A	0	2
Legislator B	2	0
Legislator C	5	0
Interest Group RED	5	0
Interest Group BLUE	0	4

QUESTION 1

IG RED tries to ensure that BLUE is not able to gain such support, but only does so if the cost are below 5 MU.

- Is it optimal for IG RED to bribe legislator B and for how much?
  - What if IG RED pays legislator B 1 MU?

QUESTION 1

IG RED tries to ensure that BLUE is not able to gain such support, but only does so if the cost are below 5 MU.

- Is it optimal for IG RED to bribe legislator B and for how much?
  - What if IG RED pays legislator B 1 MU?
  - In the second part of the game, IG BLUE will pay 4 MU to legislator B, and policy Y will be chosen.

QUESTION 1

IG RED tries to ensure that BLUE is not able to gain such support, but only does so if the cost are below 5 MU.

- Is it optimal for IG RED to bribe legislator B and for how much?
  - What if IG RED pays legislator B 2 MU?

QUESTION 1

IG RED tries to ensure that BLUE is not able to gain such support, but only does so if the cost are below 5 MU.

- Is it optimal for IG RED to bribe legislator B and for how much?
  - What if IG RED pays legislator B 2 MU?
  - In the second part of the game, IG BLUE can pay up to 4 MU to legislator B
  - Legislation B will be indifferent
  - **BUT rememeber:** *we assume that, when a legislator is indifferent, she favours policy X*

QUESTION 1

In summary:

- IG RED's optimal strategy is to bribe legislator B with 2MU
- IG BLUE can pay 4 MU to legislator B, but that would make legislator B indifferent, and she will still favour policy X
- Thefore, policy X will be implemented

QUESTION 1

Let's go back to the questions:

- **Which policy is implemented in the subgame-perfect Nash Equilibrium of the game?**
  - Policy X
- **What are the bribes that IGs offer to legislators**
  - IG RED pays 2 MU to legislator B
  - IG BLUE will not offer any bribes
- **What are player's payoffs?**
  - IG RED: 5 (-3) = 2
  - IG BLUE: 0
  - Legislator A: 0
  - Legislator B: 2 (+2) = 4
  - Legislator C: 5

QUESTION 2

(ii) Now assume that the interest group BLUE moves first and interest group RED moves second. Which policy is implemented in the subgame-perfect Nash equilibrium of the game? What are the bribes that the interest groups offer to the legislators?

Actor	Policy X	Policy Y
Legislator A	0	2
Legislator B	2	0
Legislator C	5	0
Interest Group RED	5	0
Interest Group BLUE	0	4

QUESTION 2

Applying the same reasoning as before, does IG BLUE has a viable strategy of ensuring that policy implemented is Y?

QUESTION 2

What is IG BLUE's optimal strategy?

- Bribing legislator A is not optimal. This legislator's preferred policy is Y

Actor	**Policy X**	**Policy Y**
Legislator A	0	2
Legislator B	2	0
Legislator C	5	0
Interest Group RED	5	0
Interest Group BLUE	0	4

Let's see whether it's optimal to bribe legislator C.

QUESTION 2

What is IG BLUE's optimal strategy?

- Bribing legislator A is not optimal. This legislator's preferred policy is Y
- Bribing legislator C is not optimal as it is too costly to make her change her policy preference. Her payoff exceeds IG BLUE's willingness to pay

Actor	**Policy X**	**Policy Y**
Legislator A	0	2
Legislator B	2	0
Legislator C	5	0
Interest Group RED	5	0
Interest Group BLUE	0	4

QUESTION 2

What is IG BLUE's optimal strategy?

- IG BLUE needs to target the cheapest legislators but also needs both legislators, A and B, to obtain the same payoff. **But do they need to attain the same payoff?**

QUESTION 2

What is IG BLUE's optimal strategy?

- IG BLUE needs to target the cheapest legislators but also needs both legislators, A and B, to obtain the same payoff. **But do they need to attain the same payoff?**
  - Because IG RED will target the cheapest legislator in the second round, and policy X will be implemented.

QUESTION 2

What if IG BLUE offers the following:

- Bribe to Legislator A: 0 MU
- Bribe to Legislator B: 4 MU

The legislator’s payoffs will be following:

- Legislator A: 0
- Legislator B: 2

In the second round, IG RED then can:

- Bribe Legislator B with 2 MU,
- Legislator B is now indifferent
- Legislator B and C vote for X
- Policy X is chosen.

QUESTION 2

IG BLUE needs to offer at least:

- Bribe to Legislator A: 3 (+ Δ) MU
- Bribe to Legislator B: 7 (+ Δ) MU

The legislators's payoffs will be following:

- Legislator A: 5 (+ Δ)
- Legislator B: 5 (+ Δ)

In the second round, IG RED then can:

- Bribe Legislator B: 5 MU ($2 + 5 < 7 + \Delta MU$)
- Legislators A and B vote for policy Y

QUESTION 2

Going back to the questions:

- **Which policy is implemented in the subgame-perfect Nash equilibrium of the game?**
  - Policy X
- **What are the bribes that the interest groups offer to the legislators?**
- No bribes are offered.
- **What are actors' payoffs?**
  - IG RED: 5
  - IG BLUE: 0
  - Legislator A: 0
  - Legislator B: 2
  - Legislator C: 5